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SPATIAL TASK OF INTERACTION OF SEISMIC WAVES ON THE CYLINDRICAL SHELL LOCATED IN A VISCOUS ENVIRONMENT

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Abstract
The infused work is devoted to the impact of harmonic waves on spatial cylindrical shell. The main goal is to develop a theoretical basis and generalize the mathematical model for assessing dynamic behavior of cylindrical shells, which are in interaction with deformable medium under wave effects. The research uses methods of mathematical physics equations. The practical value of the work is to develop the methodology and algorithm for calculation (dynamic stress-strain state) and optimize dynamic properties of the mechanical system as a whole.

Keywords: Harmonic waves, cylindrical shell, diffraction, stress – strain state, voltage.

Introduction

At present, there exist a number of problems related to the dynamics of dissipative systems in an unbounded elastic medium, the solution of which makes it possible to reveal new patterns of oscillations and wave diffraction. Issues of diffraction theory of seismic waves, formulated as boundary-value problems of continuum mechanics, have been considered in many works [1,2,3] and applied to the analysis of underground structures.
For practical solution of such problems, a number of assumptions are introduced. In particular, when analyzing a pipeline in the transverse direction, the problem is reduced to a plane problem for a stationary wave. Further simplifications

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involve reducing the diffraction problem to a quasi-static one, i.e., to an ordinary static problem with external boundary conditions that take into account seismic effects in the form of a constant coefficient. For example, in [4,5] a solution was obtained for estimating contact seismic pressures on a reinforced cavity under the action of a plane stationary seismic wave. In this case, the deformations of the seismic wave upon encountering a cavity are not taken into account.

This method was extended to transportation tunnels in [6,7] for the plane containing a reinforced ring, where the external boundary conditions (at infinity) were adopted according to the formula for engineering seismic pressure. The solution obtained in [6] showed that the intensity of contact pressures (seismic pressure on the tunnel) largely depends on the stiffness ratio of the surrounding rock and the lining, as well as on the geometry of the lining itself. In [8] it was noted that the solutions obtained in [9] are valid for the case of relatively deep tunnel embedment. The same considerations are also highlighted in [10,11].

In the present work, the problem of wave incidence on a spatial shell is investigated.

2. Problem Statement

This problem is considered in a cylindrical coordinate system r, θ, z. The medium, the layer, and the shell are assumed to be homogeneous and isotropic. The displacements at the interface between the medium and the shell take the following form:

$$U_r^{pp2} = M \cdot \sin\theta_2 \cdot \alpha_1 \cdot H^{(2)/}(\alpha_1 r_1) - L \cdot \cos^2\gamma_2 \cdot \frac{i \cdot \omega}{C_{S1}} \cdot H_n^{(2)}(\Omega_1 r_1) +$$

$$\frac{1}{r} \cdot K \cdot \cos\gamma_2 \cdot i \cdot n \cdot H_n^{(2)}(\beta_1 r_1)$$

$$U_\theta^{pp2} = M \cdot \sin\theta_2 \cdot \frac{1}{r} \cdot H_n^{(2)}(\alpha_1 r_1) - K \cdot \cos\gamma_2 \cdot \beta_1 \cdot H_n^{(2)/}(\beta_1 r_1)$$

$$U_z^{pp2} = -M \cdot \sin\theta_2 \cdot \frac{i\omega \cdot \cos\theta_0}{Cp_1} \cdot H_n^{(2)/}(\alpha_1 r) + L \cdot \cos\gamma_1 \cdot \left(\Omega_1 \cdot H_n^{(2)/}(\Omega_1 r) + H_n^{(2)}(\Omega_1 r)\right)$$

The corresponding stresses at the interface with the shell are expressed as follows:

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$$\frac{1}{2\mu}G_{rz}^{(pp2)} = M\cdot\sin\theta_1\left(\begin{array}{l}\frac{1-\nu_1}{1-2\nu_1}\alpha_1^2\cdot H_n^{(2)//}(\alpha_1 r_1)-\frac{\nu_1}{r_1^2(1-2\nu_1)}\cdot n^2\cdot H_n^{(2)}(\alpha_1 r_1)+\\ +\frac{\nu}{r(1-2\nu)}\cdot\alpha_1\cdot H_n^{(1)//}(\alpha_1 r)\end{array}\right)+$$

$$L\cdot\cos\gamma_1\cdot\left(\frac{i\omega\cos\gamma_1}{c_{s1}}\cdot\Omega_1\cdot H_n^{(2)/}(\Omega_1 r_1)+\frac{1}{r_1}\cdot\frac{i\omega\cos\gamma_1}{c_{s1}}\cdot H_n^{(2)}(\Omega_1 r_1)\right)$$

$$G_{r\theta}^{pp2} = M\cdot\sin\theta_2\cdot\frac{2in}{r_1}\cdot\left(\alpha_1\cdot H_n^{(2)/}(\alpha_1 r_1)-\frac{1}{r_1}\cdot H_n^{(2)}(\alpha_1 r_1)\right)+$$

$$+L\cdot\cos^2\gamma_2\cdot\frac{n\cdot\omega}{r_1\cdot C_{s1}}\cdot H_n^{(2)}(\Omega_1 r_1)-K\cdot\cos\gamma_2\left(\frac{n^2}{r_1^2}\cdot H_n^{(2)}(\beta_1 r_1)+\right.$$

$$\left.+\beta^2{}_1\cdot H_n^{(2)//}(\beta_1 r_1)-\frac{\beta_1}{r_1}\cdot H_n^{(2)/}(\beta_1 r_1)\right)$$

$$G_{rr}^{pp2} = M\cdot\sin\theta_2\cdot\left(\begin{array}{l}\frac{1-\nu_1}{1-2\nu_1}\cdot\alpha^2{}_1\cdot H_n^{(2)//}(\alpha_1 r_1)-\frac{\nu_1 n^2}{r_1^2(1-2\nu_1)}\cdot H_n^{(2)}(\alpha_1 r_1)+\\ +\frac{\nu_1\alpha_1}{r(1-2\nu_1)}\cdot H_n^{(2)/}(\alpha_1 r_1)-\frac{\nu_1}{1-2\nu_1}\cdot\frac{\omega^2\cos^2\theta_2}{C_{p_1}^2}\cdot H_n^{(2)}(\alpha_1 r_1)\end{array}\right)+$$

$$+L\cdot\cos\gamma_2\cdot\frac{\Omega_1 i\cdot\omega}{r\cdot C_{p_1}^2}\cdot H_n^{(2)/}(\Omega_1 r_1)+K\cdot\cos\gamma_2\left(\frac{in}{r}\cdot\beta_1\cdot H_n^{(2)/}(\beta_1 r_1)-\frac{in}{r^2}\cdot H_n^{(2)}(\beta_1 r_1)\right)$$

External loads acting on the shell from the layer:

$$\begin{pmatrix}q_1\\ q_2\\ q_3\end{pmatrix}=\frac{r_0(1-\nu_0^2)}{Eh_0}(\rho_0 h_0 w^2)\begin{pmatrix}-G_{rz}^{pp2}\\ -G_{r\theta}^{pp2}\\ -G_{rr}^{pp2}\end{pmatrix}$$

Boundary conditions at the layer–medium interface:”

at $r = r_0$;

$u_{r1} = u_{r0}$; $\sigma_{rr1} = \sigma_{rr0}$; $u_{\theta 1} = u_{\theta 0}$;

$\sigma_{r\theta 1} = \sigma_{r\theta 0}$; $u_{z1} = u_{z0}$; $\sigma_{rz1} = \sigma_{rz0}$.

Boundary conditions for the shell with a layer:

at $r = r_1$;

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$U_r^{pp2} = u_r - u_r^{(p)}; U_\theta^{pp2} = v_\theta - u_\theta^{(p)}; U_z^{pp2} = w_z - u_z^{(p)}$

We obtain a complex algebraic system.

$$[C](A,B,C,D,E,F,M,L,K)^{T} = [p]$$

3. Results and analysis

By solving the algebraic system, we obtain A,B,C,D,E,F,M,L,K and determine the hoop (circumferential) and axial forces of the shell.

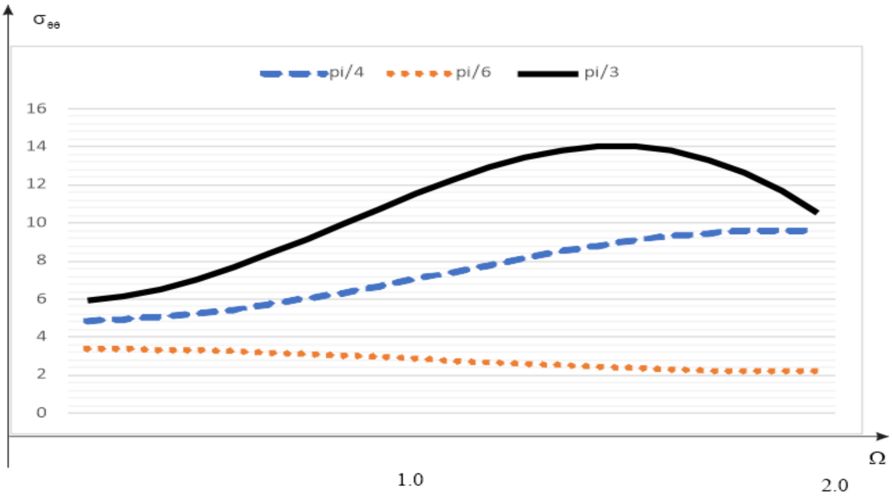

Fig. 1. Hoop (circumferential) stresses of the cylindrical layer: medium – soft soil, shell – steel, at 60^0, Ω=1.4, $G_{\theta\theta}$=13.87; at 45^0, Ω=2, $G_{\theta\theta}$=6.64; at 30^0, Ω=0.1, $G_{\theta\theta}$=3.39

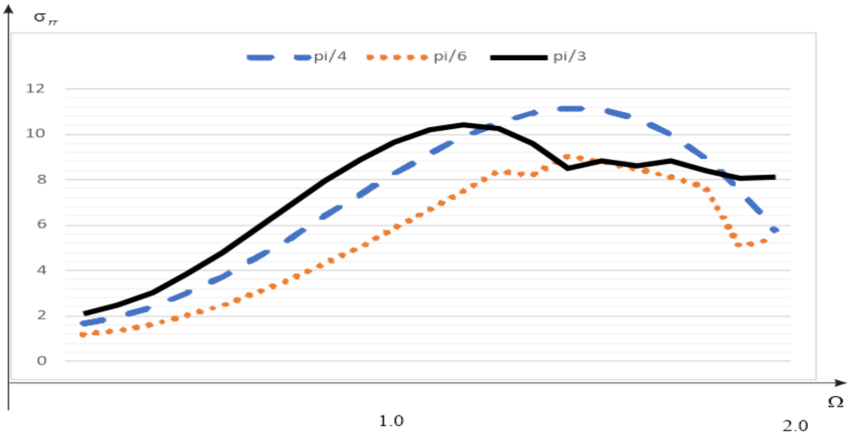

Fig. 2. Radial stresses of the cylindrical layer: medium – soft soil, shell – steel, at 60^0, Ω=1.2, G_{rr}=10.43; at 45^0, Ω=1.5, G_{rr} =11.14; at $30^{0,}$ Ω=1.4, G_{rr}=9.19

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$$N_z=\frac{Eh_0}{1-v_0^2}\left(\begin{array}{l}\dfrac{i\omega\cos\theta_2}{C_p}\cdot(M\cdot\sin\theta_2\cdot H_n^{(2)/}(\alpha_1 r_1)\cdot\alpha_1-L\cdot\cos^2\gamma_2\cdot\dfrac{i\cdot\omega}{C_{S0}}\cdot H_n^{(2)}(\Omega_1 r_1)\\ +K\cdot\dfrac{1}{r}\cdot\cos\gamma_2\cdot i\cdot n\cdot H_n^{(2)}(\beta_1 r_1))-\dfrac{v_0}{r_0}(M\cdot\sin\theta_2\dfrac{i\omega\cdot\cos\theta_2}{Cp}\cdot H_n^{(2)}(\alpha_1 r_1)\\ +L\cdot\cos\gamma_2\left(\Omega_1\cdot H_n^{(2)/}(\Omega_1 r_1)+\dfrac{1}{r_1}H_n^{(2)}(\Omega_1 r_1)\right)+\varphi_0\sin\theta_0\cdot\alpha J^{/}(\alpha r)\end{array}\right)$$

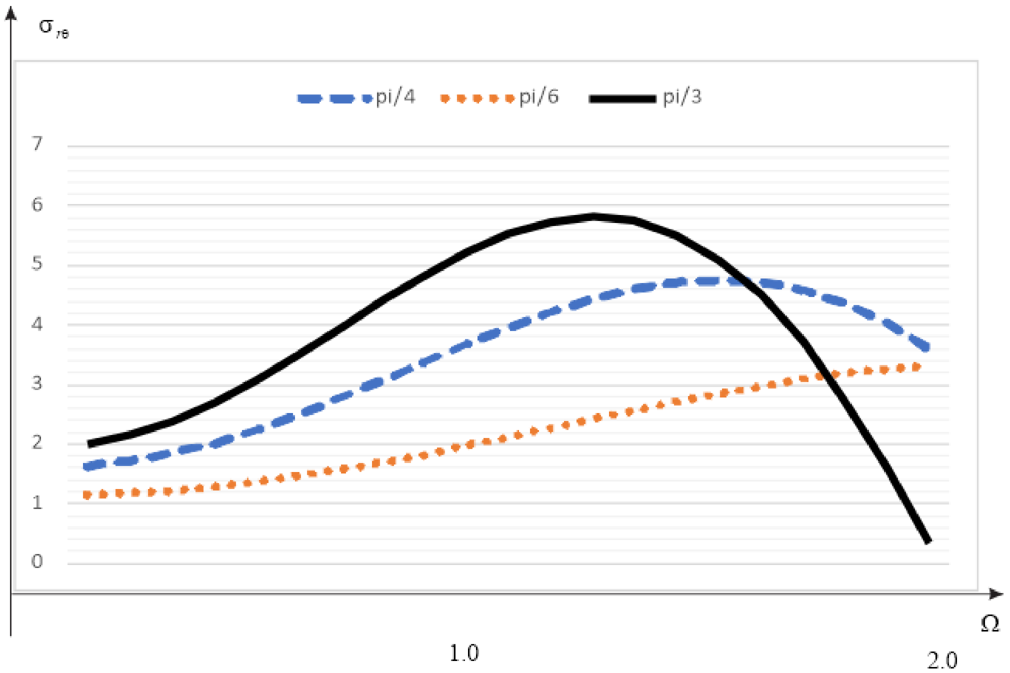

Fig. 3. Tangential stresses of the cylindrical layer: medium – soft soil; shell – steel, at 60^0, Ω=1.3, G_{rr}=5.82; at 45^0 ,Ω=1.6, G_{rr} =4.75; at 30^0, Ω=1.9,G_{rr} =3.26

$$N_\theta=\frac{Eh_0}{1-v_0^2}\left(\begin{array}{l}\dfrac{iv\omega\cos\theta_0}{C_p}\cdot(M\cdot\sin\theta_2\cdot H^{(2)/}(\alpha_1 r_1)\cdot\alpha_1-L\cdot\cos^2\gamma_1\cdot\dfrac{i\cdot\omega}{C_S}\cdot H_n^{(2)}(\Omega_1 r_1)\\ +K\cdot\dfrac{1}{r_1}\cdot\cos\gamma_1\cdot i\cdot n\cdot H_n^{(2)}(\beta_1 r_1))-\dfrac{1}{r_1}(M\cdot\sin\theta_2\dfrac{i\omega\cdot\cos\theta_2}{Cp}\cdot H_n^{(2)}(\alpha_1 r_1)\\ +L\cdot\cos\gamma_2\left(\Omega_1\cdot H_n^{(2)/}(\Omega_1 r_1)+\dfrac{1}{r_1}H_n^{(2)}(\Omega_1 r_1)\right)-\varphi_0\sin\theta_0\cdot\dfrac{i\omega\cdot\cos\theta_0}{Cp}J(\alpha r)\end{array}\right)$$

When a viscoelastic layer is considered, with the medium and the shell being elastic, we take a three-parameter kernel as the relaxation kernel:

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$$R(t) = \frac{A_n e^{-\beta t}}{t^{1-\alpha}}$$

with the parameters:

$A_1 = 0.048; \beta = 0.05; \alpha = 0.1;$

$A_1 = 0.078; \beta = 0.05; \alpha = 0.1$ — high viscosity.

In the case of solving the stationary problem, cosine and sine forms are used for the kernel:

$$\Gamma^C(\omega) = \frac{A\Gamma(x)}{(\omega^2 n^2 + \beta^2)^{1/2}} \cos(\alpha arctg \frac{\omega n}{\beta}),$$

$$\Gamma^s(\omega) = \frac{A\Gamma(x)}{(\omega^2 n^2 + \beta^2)^{1/2}} \sin(\alpha arctg \frac{\omega n}{\beta}),$$

where $\Gamma(\omega)$ is the gamma function and ω is the real frequency.

Γ(0.1) = Γ(0.1+1)/0.1=Γ(1.1)/0.1, Γ(1.1) = in {n = 0} = 0.9514

Γ(1.1) = npu {n = 1} = 0.9509, $E = E_0(1 - \Gamma^c(\omega) - i\Gamma^s(\omega))$ taking viscosity into account.

The results of force calculations in the shell (spatial problem), embedded in a viscoelastic medium and interacting with seismic waves incident on a cylindrical layer and the shell, under the action of longitudinal (or transverse) waves, are presented in Figures 1–3. The figures show that the maximum force factors in the shell appear in the low-frequency range under the action of transverse waves.

Thus, we carried out a three-dimensional analysis of cylindrical shells (pipelines) subjected to seismic wave action. At low frequencies, expressions were obtained for estimating the amplitude of stresses in deeply buried pipelines for a given wavelength $\Omega = \omega R / C_2$. It is shown that the induced maximum stresses are governed by the direction of incident waves and, primarily, by the elastic moduli of the medium and the cylinder. In the design of underground structures, the importance of considering spatial factors was established. In addition, the frequency response function (FRF) of the shell (ring loads A_0) was studied for a concrete shell in soft soil ($\theta = \frac{\pi}{2}, \theta_0 = 90^0$) under the action of longitudinal waves.

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Conclusion

To describe the dissipative properties of the system as a whole, the concept of a "global damping coefficient" is introduced. In the case of a structurally homogeneous mechanical system, the global damping coefficient is entirely determined by the imaginary part of the first (by modulus) complex phase velocity. In the case of a structurally non-homogeneous mechanical system, the imaginary parts of both the first and the second natural frequencies serve as the global damping coefficient, depending on the values of the geometric parameters. It has been established that the optimal damping of oscillations in non-homogeneous systems occurs when the real parts of the phase velocities of different modes are close (or converge), and the corresponding imaginary parts of these phase velocities intersect at this point (or become equal). In this case, both vibration modes of the mechanical system provide the same energy dissipation.

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