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UNIVERSAL APPLICATION OF FOURIER SERIES IN ENGINEERING, MEDICINE, AND ECONOMICS

Alisher Omonov
Lecturer, Department of Higher and Applied Mathematics
Tashkent State University of Economics
alisher.omonov1992@mail.ru

Javlon Fayziyev
Lecturer, Department of Higher and Applied Mathematics
Tashkent State University of Economics
javlonabduvoxidovich@gmail.com

Oltinbek Mirzamurodov
Lecturer, Department of Higher and Applied Mathematics
Tashkent State University of Economics
mirzamurodovoltinbek15@gmail.com

Abstract:
In today's era of digital technologies, signal and image processing is widely applied in various fields such as medicine, communications, audio and video technologies, artificial intelligence, and engineering. In these processes, Fourier analysis—namely, Fourier series and integral transforms—serves as one of the fundamental mathematical tools for analyzing the composition of signals and images, compressing them, and filtering noise. With the help of Fourier series and transforms, complex functions can be decomposed into sine and cosine waves, which facilitates easier analysis, noise reduction, and data compression. This article provides a detailed analysis of the fundamental concepts of Fourier series and transforms, their mathematical foundations, and their practical applications across various fields.

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Keywords: Fourier series, Fourier transform, integral transforms, medical imaging and Fourier analysis, biometric identification, artificial intelligence and Fourier analysis

FURYE QATORLARINING MUHANDISLIK, TIBBIYOT VA IQTISODIY SOHALARIDA UNIVERSAL QO‘LLANILISHI

Omonov Alisher Toshpo'lat o'g'li
Toshkent davlat iqtisodiyot universiteti
“Oliy va amaliy matematika” kafedrasi o‘qituvchisi
alisher.omonov1992@mail.ru

Fayziyev Javlon Abduvoxidovich
Toshkent davlat iqtisodiyot universiteti
“Oliy va amaliy matematika” kafedrasi o‘qituvchisi,
javlonabduvoxidovich@gmail.com

Mirzamurodov Oltinbek Sultonali o‘g‘li
Toshkent davlat iqtisodiyot universiteti
“Oliy va amaliy matematika” kafedrasi o‘qituvchisi,
mirzamurodovoltinbek15@gmail.com

Annotatsiya

Hozirgi raqamli texnologiyalar davrida signal va tasvirlarni qayta ishlash turli sohalarda, jumladan, tibbiyot, aloqa, audio va video texnologiyalar, sun’iy intellekt hamda muhandislikda keng qo‘llaniladi. Ushbu jarayonlarda Furye tahlili, ya’ni Furye qatorlari va integral almashtirishlari, signal va tasvirlarning tarkibini aniqlash, ularni siqish va filtrlash kabi muhim vazifalarni bajarishda asosiy matematik usullardan biri hisoblanadi.Furye qatorlari va integral almashtirishlar yordamida murakkab funksiyalarni sinus va kosinus to‘lqinlariga ajratish mumkin. Bu esa signal va tasvirni osonroq tahlil qilish, shovqinlardan tozalash va ma’lumotlarni siqish imkonini beradi. Ushbu maqolada Furye qatorlari va integral almashtirishlarning asosiy tushunchalari, ularning matematik asoslari va amaliy qo‘llanilish sohalari haqida batafsil tahlil beriladi.

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Kalit so'zlar: Furye qatorlari, Furye transformasi, integral almashtirishlar, tibbiy tasvirlash va Furye tahlili, biometrik identifikatsiya, sun'iy intellekt va Furye tahlili

Introduction

In today's era of digital technologies and artificial intelligence, the rapid growth in data volume and the need for its analysis have significantly increased the demand for advanced mathematical methods. In particular, precise, reliable, and efficient mathematical tools are essential in fields such as signal and image processing, compression, filtering, and analysis. In this context, Fourier series and Fourier integral transforms stand out as fundamental tools of critical importance. Although these mathematical approaches may initially appear complex, they are distinguished in practice by their ability to yield clear and effective results.

Fourier analysis is a method that determines the frequency spectrum of complex functions by decomposing them into sine and cosine waves. This technique focuses not on how a signal changes over time, but on analyzing the frequencies contained within it. The method is primarily applied in two forms: Fourier series for periodic functions and Fourier integral transforms for non-periodic functions. Both approaches are theoretically well-established and widely used in various disciplines, including physics, engineering, optics, medicine, telecommunications, and artificial intelligence.

Notably, medical diagnostics—especially MRI and CT technologies—rely heavily on Fourier analysis to provide accurate and comprehensive internal images of the human body. Likewise, in the field of artificial intelligence, Fourier transforms play a crucial role in facial recognition, biometric identification, and the automated analysis of voice and image data. In digital signal processing, the Discrete Fourier Transform (DFT) and its optimized version, the Fast Fourier Transform (FFT), enable the efficient real-time processing of large datasets.

In recent years, wavelet transforms have also gained popularity as an alternative to Fourier transforms. These allow for simultaneous analysis in both time and frequency domains, making them particularly effective for working with complex and dynamic signals.

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This article provides an in-depth analysis of the theoretical foundations, formulas, and practical application domains of Fourier series and integral transforms, as well as their integration with modern technologies. It also highlights the advantages of these approaches in strategic fields such as medical diagnostics, engineering, artificial intelligence, and digital communication.

Fourier series are series that represent periodic functions as an infinite sum of sine and cosine and are based on the following basic formula.

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cdot\cos\frac{\pi nx}{L}+b_n\cdot\sin\frac{\pi nx}{L})$$

Here a_n and b_n Fourier coefficients can be calculated as follows.

$$a_n=\frac{1}{L}\int_{-L}^{L}f(x)\cdot\cos\frac{\pi nx}{L}dx \;;\quad b_n=\frac{1}{L}\int_{-L}^{L}f(x)\cdot\sin\frac{\pi nx}{L}dx$$

L is the half-period of the function.
If the function $f(x)$ is not periodic, it cannot be expanded into a Fourier series. In such cases, the Fourier integral transform is used, i.e., the function is represented on an infinite interval (through continuous frequencies instead of integers) and is expressed using the following formula.

$$F(\omega)=\int_{-\infty}^{\infty}f(x)\cdot e^{-i\omega x}dx\,,$$

Here, i is a complex number, and ω represents the rate of change of the signal over time.
Accordingly, while the expansion into Fourier series results in discrete frequencies, the Fourier integral transform produces continuous frequency spectra.
Due to these characteristics, Fourier series and integral transforms are widely applied in many areas of our lives, including music and audio processing, electrical engineering, mechanics, telecommunications, medicine, and other fields.
Below, the application of these methods in specific domains is examined in detail.

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Medicine

Fourier series and integral transforms play an especially important role in medicine, particularly in imaging diagnostics and the analysis of medical signals. Medical imaging technologies such as Magnetic Resonance Imaging (MRI) and Computed Tomography (CT) are based on these mathematical methods. In MRI systems, radio frequency signals are transmitted into the human body, and their echoes are recorded. These sets of signals are unsuitable for direct processing and are converted into the frequency domain using Fourier integral transforms. This allows for the formation of images based on the electromagnetic properties of various tissues. As a result, physicians are able to detect diseases or pathological changes within the human body.

Engineering and Physics

The Fourier transform is used in engineering and physics for modeling waves and processing signals. In optics, this method is applied to analyze light diffraction and interference and is used in reconstructing images through lenses and optical filters. For example, images obtained via telescopes or microscopes may be distorted due to atmospheric noise or optical aberrations; however, using the Fourier transform, these images can be cleaned and restored to a clearer state.

Artificial Intelligence and Digital Image Processing

The combination of Fourier series and artificial intelligence enables facial detection and biometric identification. This technology is widely used in security systems, banking authentication, and by law enforcement agencies for identity verification. The working principle of such systems is based on frequency-domain analysis of the image and its classification using artificial intelligence. Initially, the human face is scanned using cameras or infrared sensors, and the resulting image is cleaned of noise and adjusted for lighting balance. Usually, the image is presented in the form of a pixel matrix; however, because direct analysis is complex, the image is converted into the frequency domain using the Fourier transform.

As a result, the high frequencies of the image represent fine details—such as wrinkles, textures, and contours—while the low frequencies describe the main

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structural elements—such as eyes, nose, lips, and face shape. The key components in the frequency spectrum of the face are then extracted and analyzed using artificial intelligence algorithms. Neural networks compare these features with the existing biometric database. If the facial image matches an existing individual in the database, the system confirms the identity; otherwise, the user may be registered as a new person, or authentication may be denied.

Extracting the important features of the face using the Fourier transform not only reduces the amount of data but also accelerates the analysis process.

In the field of modern signal processing, Fourier series occupy a special place.

The expansion of a periodic signal $f(t)$ into a Fourier series is given by:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{2\pi nt}{T}\right) + b_n \sin\left(\frac{2\pi nt}{T}\right) \right]$$

Here, the coefficients are:

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos\left(\frac{2\pi nt}{T}\right) dt, \quad b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin\left(\frac{2\pi nt}{T}\right) dt$$

Here, the coefficients are calculated using the following formulas.

In practical signal processing tasks, especially in the design of digital filters, Discrete Fourier Transforms (DFT) are widely used.

The Fast Fourier Transform (FFT) algorithm reduces the computational complexity from $O(N^2)$ to $O(N \log N)$, which is highly important for processing large datasets in real-time applications.

In addition to Fourier integral transforms, wavelet transforms have also gained significant importance in recent years. Unlike Fourier transforms, wavelet transforms allow for the local analysis of signals in both the time and frequency domains. A wavelet transform is expressed in the following form:

$$W_{\psi} f(a,b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} f(t) \psi^* \left(\frac{t-b}{a}\right) dt$$

Here, $\psi(t)$ is the mother wavelet, a is the scale parameter, and b is the shift parameter.

Wavelet transforms play an important role in many practical tasks such as image processing, signal compression, noise reduction, and feature extraction.

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For example, the JPEG2000 image compression standard is based on wavelet transforms and provides higher efficiency than JPEG, especially at low bit rates.

Thus, Fourier series and integral transforms have become one of the methodological foundations of modern science and technology. They are of fundamental importance not only for theoretical research but also for practical applications. In particular, medical diagnostic systems such as Magnetic Resonance Imaging (MRI) and Computed Tomography (CT) are based on Fourier transforms and allow the acquisition of highly accurate data on human tissue. In the field of telecommunications, Fourier analysis forms the basis for efficient signal encoding, multiplexing, and modulation. Modern communication technologies such as Orthogonal Frequency Division Multiplexing (OFDM) rely on this theory.

In engineering and technical fields, Fourier transforms are used for vibration analysis, structural mechanics, and modeling of heat transfer processes. In artificial intelligence and machine learning algorithms, they are widely used to form feature vectors of signals and improve image and speech recognition systems. In economics and the financial sector, Fourier analysis serves as an effective tool for studying time series, identifying market cycles and trends, and forecasting future events. This significantly enhances the efficiency of business process optimization and strategic decision-making.

The universality and adaptability of these mathematical methods enable their application in solving complex problems across many industries. As a result, they contribute significantly to resource savings, increased production efficiency, and the advancement of technological innovations. Modern scientific and technological progress largely depends on the practical application of these theoretical foundations.

Conclusion

In conclusion, Fourier series and integral transforms have become an integral part of modern science and technology. Through them, many practical tasks such as the analysis, compression, denoising, identification, and modeling of complex functions and signals are solved. These methods play an irreplaceable role in efficiently processing large volumes of data — one of the core requirements of the digital era.

In the field of medicine, MRI and CT technologies based on Fourier transforms enable doctors to detect diseases at an early stage. In engineering and physics, they are used for modeling waves and vibrations, and for optimizing devices and systems. In artificial intelligence, they serve as a key tool in identifying frequency characteristics for facial recognition, biometric authentication, and data compression.

The universal and practical nature of Fourier analysis allows it to be applied in technical, medical, economic, and information technology fields. For instance, modern mobile communication systems (4G, 5G) built on OFDM technology are directly based on the Fourier transform. Additionally, this method yields effective results in economic forecasting and the analysis of trends and cycles.

Therefore, in-depth study of Fourier series and integral transforms should be considered not only within the realm of mathematical knowledge, but also as a key component of modern technological development. Future scientific research and innovations will open new domains based on these mathematical foundations. Their application in practice will contribute not only to science, but to the advancement of society as a whole.

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