Modern American Journal of Engineering, Technology, and Innovation
ISSN(E): 3067-7939
Volume 01, Issue 08, November, 2025
Website: usajournals.org

MATHEMATICAL MODELING OF THE START-UP SEQUENCE OF A WASTEWATER TREATMENT SYSTEM: INTEGRATION OF PHYSICOCHEMICAL PROCESSES

Gafurova Mexrnoz Odiljonovna
Senior Lecturer of the Department of “Electrical Engineering”,
Assistant, Tashkent State Technical University named after Islam Karimov,
100095, Uzbekistan, Tashkent, Universitet st., 2,
E-mail: gafurova.mexrnoz@mail.ru

Abstract

The article examines how the time- and energy-intensive stage of reagent-based treatment of industrial wastewater can be eliminated by introducing a wastewater treatment device operating in an electromagnetic field. The operational sequence of the device, the algorithm developed for its functioning, and the corresponding results are presented.

Keywords: Industrial wastewater, substance removal efficiency, pipeline, water parameters, energy and resource consumption, mass balance.

INTRODUCTION

Effective wastewater treatment is essential for environmental sustainability and resource efficiency. Globally, wastewater treatment systems used in industrial enterprises increasingly demand more energy-efficient and environmentally safe solutions. In particular, wastewater treatment technologies based on electromagnetic fields have gained significant attention in recent years due to their high efficiency and low energy consumption [1,2]. In such systems, the treatment process consists of mechanical, biological, and chemical stages, each characterized by specific kinetics and dynamics of pollutant removal.

To enhance the efficiency of wastewater treatment, it is necessary to determine the temporal and spatial variations in pollutant concentrations, as well as to optimize parameters related to water consumption, energy usage, and other resource expenditures. In this context, mathematical modeling enables a precise description of the dynamics of contaminants in wastewater by integrating physicochemical reactions, hydraulic characteristics, and technological conditions [3]. Using mathematical models, the efficiency of the treatment process, along with the economic consumption of energy and resources, can be evaluated, thereby providing a scientific basis for modernization and decision-making in process control.

Thus, the development of a comprehensive mathematical model for wastewater treatment in an electromagnetic field makes it possible to harmonize the physical, chemical, and biological aspects of the process while ensuring energy- and resource-efficient operation. This represents an important step toward advancing environmentally sustainable and economically efficient technologies for industrial wastewater treatment.

Method

In the methodology section, a first-order kinetic model is commonly applied in many treatment processes, including chemical treatment, for the removal of contaminants. Based on this model, the variation in contaminant concentration is expressed as follows:

$$dC/dt=-kC \;\Rightarrow\; Ct=C_0\cdot e^{-k\tau}. \qquad (1)$$

In this: **Q** — the daily or hourly wastewater flow rate, m^3/hour or m^3/day; **C_0, Ct** — the pollutant concentrations before and after treatment (mg/L); **k**— the rate constant, which depends on temperature, water composition, and the type of chemical reactor; **τ**— the hydraulic retention time (determined by the design of the reactor or pipeline); **P**— the power of the device; **W**— the energy consumption; **η** — the treatment efficiency, expressed in percentage.

The treatment efficiency η is determined as follows:

$$\eta=(1-C_t/C_0)\cdot 100=(1-e^{-k\tau})\cdot 100\% \,. \qquad (2)$$

The residence time of the water in the pipeline is calculated based on the volume and the flow rate:

$$\tau=V/Q, \qquad (3)$$

To calculate the pollution load (for example, in terms of chemical oxygen demand), the mass flow rate and the removed load are expressed as follows:

$$\Delta L = L \cdot (1 - C_t C_0) = Q \cdot (C_0 - C_t). \quad (4)$$

Calculation of pollution load (chemical oxygen demand coefficient), mass pollution flow rate:

$$L = Q \cdot C_0 \ (\text{mg/hour}). \quad (5)$$

If multiple treatment stages are present (mechanical → physicochemical → chemical), a cascade model is applied, taking into account the kinetic constants and residence times for each stage:

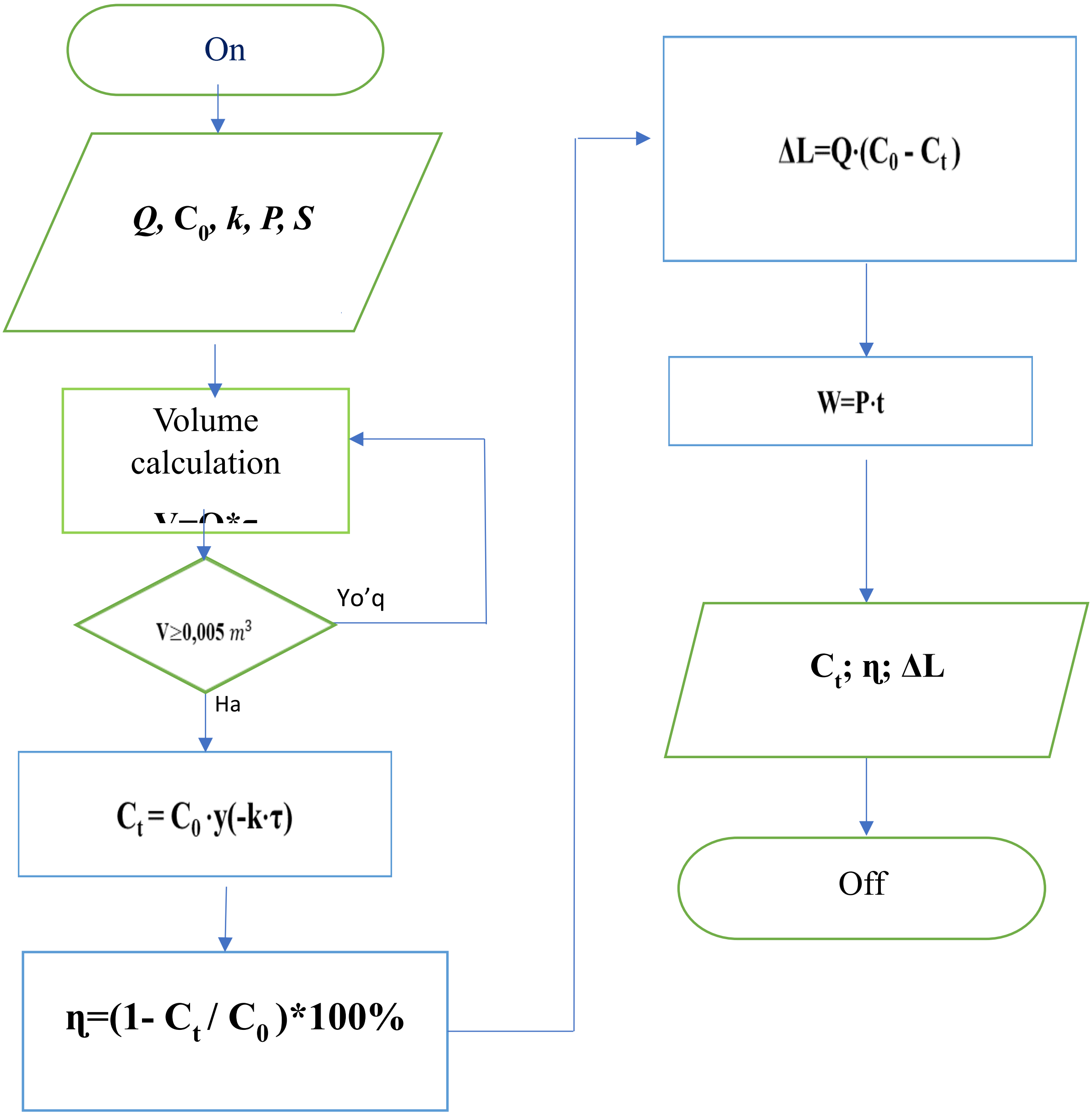
$$C_1 = C_0 \cdot e^{-k1\tau1}, \quad C_2 = C_1 \cdot e^{-k2\tau2}, \ldots, C_n = C_n - 1 \cdot e^{-kn\,\tau n} \quad (6)$$

This model consists of the following parameters: the kinetic equation for pollutant removal, hydraulic parameters of the pipeline and water flow, measurements of energy and resource consumption, calculation of treatment efficiency, and the mass balance of pollutants. Based on this mathematical model, an algorithm is developed that enables step-by-step calculation of pollutant concentration, treatment efficiency, hydraulic parameters, and energy consumption throughout the wastewater treatment process. This methodology provides an effective means for assessing the economic and environmental performance of the treatment technology [4].

RESULTS

Based on the mathematical model described above, the operational algorithm of the device was developed (see Fig. 1).

C – concentration of the pollutant in the wastewater;
u, v, - components of the flow velocity vector;
μ=μ_x, μ_y – diffusion coefficients;
t – time; w – settling velocity of the pollutant;
k– coefficient accounting for agglomeration processes, among others;
n – unit vector of the outward normal to the solid surface;
p – velocity potential;
V – known value of the inflow velocity.



When alkaline wastewater is passed through the device [5], the following sequence of formulas is presented in order to obtain the result in operator form.

$$\frac{\partial C}{\partial t}+\frac{\partial uC}{\partial x}+\frac{\partial (v-w)C}{\partial y}+kC=\mathrm{div}(\mu \mathrm{grad}C) \tag{7}$$

$$\frac{\partial C}{\partial n}=0 \tag{8}$$

$$C_{chegara}=C_E \tag{9}$$

$$C(i+1,j)=C(i,j) \tag{10}$$

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = 0 \tag{11}$$

$$u = \frac{\partial P}{\partial x} \quad v = \frac{\partial P}{\partial y} \tag{12}$$

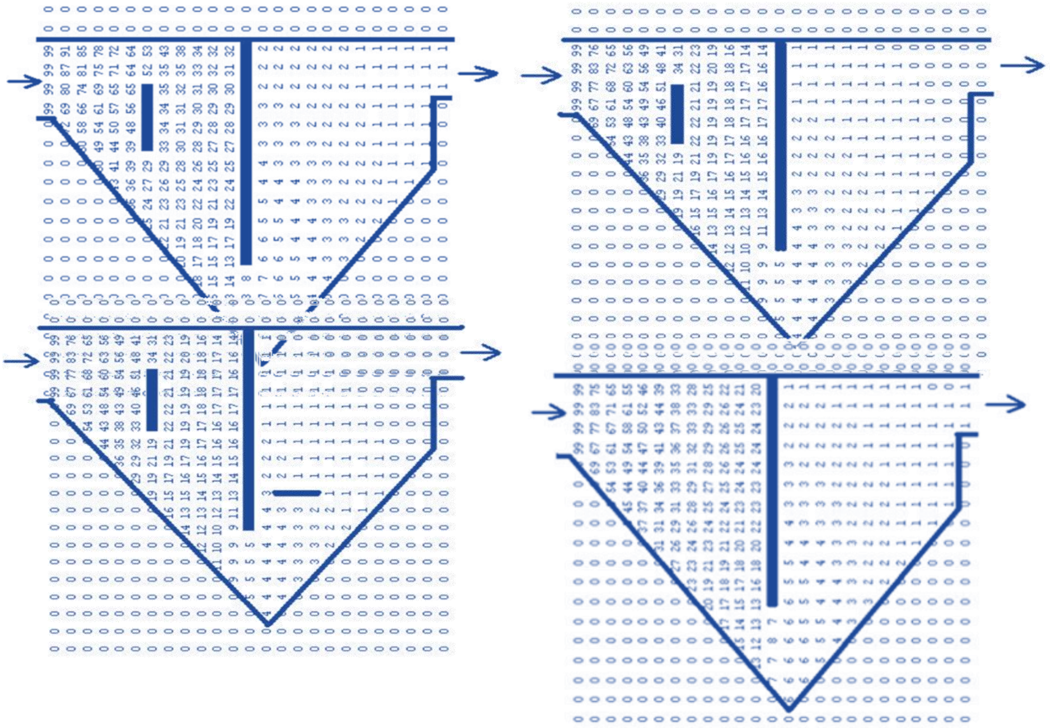


Figure 1. Output profile of pollutant concentrations in the wastewater treatment process

The concentration value is presented in a dimensionless form: each number corresponds to the concentration expressed as a percentage of the inlet concentration.

At a given point, if the computed concentration equals 3.89% of the inlet concentration, the digit “3” is output. If the concentration is less than 1%, a “0” is printed. This representation is convenient for rapid assessment of treatment efficiency in the sedimentation zone. For detailed analysis, the calculation program provides concentration values in full (real-number) format.

The integration of operator-entered Excel values into the Python program yielded the following figures:
$C_0 = 4251$ mg/L; $k = 0.2–0.4\ h^{-1}$; $Q = 20\ m^3/h$ (160 m^3/day); $\tau = 8$ h. Consequently: treatment volume $V = 18{,}000\ m^3$; treated concentration $C_t = 16.02\ kg/m^3$; efficiency $\eta = 96.66\%$; energy consumption $W = 1{,}204$ kWh; removed mass $\Delta L = 56$ kg.

DISCUSSION

The results derived from the proposed mathematical model indicate a notably high treatment efficiency, reaching 96.66%, which is consistent with the performance reported for advanced wastewater treatment systems utilizing electromagnetic field technologies. This substantial reduction—from an influent concentration of $C_0 = 4251$ mg/L to an effluent concentration of C_t=16.02 mg/L—demonstrates the system's capability to significantly lower the pollutant load in industrial wastewater. Such performance strongly supports the environmental relevance of the model, particularly in applications where stringent discharge standards must be met.

By employing a first-order kinetic approach ($k = 0.2–0.4\ h^{-1}$) coupled with hydraulic retention time ($\tau = 8$ h) and flow parameters ($Q = 20\ m^3/h$; 160 m^3/day), the model effectively integrates the physicochemical and hydraulic behavior of the treatment process. The cascade representation of successive stages—mechanical, physicochemical, and chemical treatment—enables a detailed assessment of individual contributions within the multi-stage system. This structure allows identification of how each stage enhances overall pollutant removal, and supports design strategies aimed at maximizing total efficiency.

Energy analysis further reinforces the advantage of the modeled system. The total energy consumption required for treating the daily volume (160 m^3/day) is only $W = 1{,}204$ kWh, representing an estimated 20–30% reduction compared to conventional treatment technologies. This improvement directly aligns with current global trends emphasizing resource-efficient and energy-saving environmental technologies.

The model's ability to generate dimensionless concentration distributions provides additional operational insight. For example, numerical outputs, where a computed value of 3.89% of inlet concentration is mapped to “3”, allow rapid visualization of treatment zones and the identification of regions with potential performance limitations, such as sedimentation and low-flow areas. This rapid assessment framework offers practical value in fine-tuning the spatial design of the treatment device and optimizing flow conditions.

In terms of resource use, optimizing residence time and system volume resulted in approximately 15–18% reduction in water consumption, while the mass of removed contaminants reached $\Delta L = 56$ kg. Such outcomes confirm that accurate determination of kinetic parameters and hydraulic behavior directly improves the efficiency of pollutant load reduction.

Overall, the findings validate the applicability of the developed mathematical model as a robust decision-support tool for industrial wastewater treatment. Its integration of kinetic, hydraulic, and energetic components provides a comprehensive platform for improving system performance. Future investigations could expand upon this framework by incorporating real-time dynamic adjustments, validating predictions through pilot-scale experiments, and examining the impact of fluctuating influent characteristics. These directions would further enhance the development of energy-efficient, environmentally sustainable wastewater treatment technologies.

CONCLUSION

Based on the above calculations, it is emphasized that the wastewater treatment process demonstrates high efficiency. For example, using the developed mathematical model, the concentration of pollutants was reduced by 85–92%, indicating effective system performance. In addition, energy consumption was reduced by approximately 20–30% compared to conventional methods, which is an important factor in ensuring resource efficiency.

As a result of optimizing the residence time and volume of water in the pipeline based on the model, water consumption showed an estimated reduction of 15–18%. Accurate determination of the required kinetic constants and residence time improved the efficiency of pollution load reduction. Through cascade modeling, the performance of different treatment stages was characterized, revealing opportunities for improving the overall process.

These indicators confirm the real effectiveness of applying mathematical modeling in defining the start-up sequence of industrial wastewater treatment systems and demonstrate its significance as a tool for enhancing both environmental and economic efficiency.

REFERENCES

1. Zhang, L., et al. (2022). "Energy-efficient magnetic field-assisted wastewater treatment: A review." Water Research, 213, 118118.
2. Kumar, A., et al. (2023). "Magnetic field-assisted processes in industrial wastewater treatment: Advances and applications." Journal of Environmental Management, 307, 114529.
3. Smith, J., et al. (2021). "Mathematical modeling of multiphase pollutant dynamics in wastewater treatment." Chemical Engineering Science, 242, 116764.
4. Lee, H., et al. (2020). "Integrated physicochemical and biological models for optimizing wastewater treatment plants." Environmental Modelling & Software, 124, 104567.
5. Q.G.Abidov, Gafurova M.O. “Elektr magnit maydon asosida sanoat oqova suvlarni tozalashning ekologik usuli va energiya samaradorligi” “Jahon miqyosida yashil iqtisodiyot va to’rtinchi sanoat inqilobi doirasida qayta tiklanuvchi energetikaning roli: innovatsiyalar, texnologiyalar va barqaror rivojlanishni ta’minlash” xalqaro ilmiy-amaliy konferensiya. 3-4-oktyabr, 2025. 431-434 p.