Modern American Journal of Engineering, Technology, and Innovation
ISSN(E): 3067-7939
Volume 01, Issue 09, December, 2025
Website: usajournals.org

FORCED VIBRATIONS OF CYLINDRICAL BODIES

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Abstract
The article considers forced vibrations of cylindrical layers in interaction with elastic systems. The internal pressures of the cylindrical layer and the elastic system, the continuous vibrational motion under the influence of harmonic, longitudinal P waves are studied. The changes in the ring stresses and absorptions generated in the cylindrical layer and the mechanical system with respect to the wave number are studied.

KIRISH

Bugungi kunda ta'limda olib borilayotgan islohotlar, o'zgarishlar o'quvchilarda nafaqat ta'limiy, balki kasb tanlashga yo'llash, hayotiy bilimlarni rivojlantirishga qaratilgandir. Kasb tanlashga yo'llash butun pedagoglar jamoasi tomonidan hal etiladigan uzliksiz ta'limning vazifasi hisoblanadi. Yoshlarni biron kasb egasi, hunarli qilib tarbiyalashni yoshlikdan o'rgatib borishimiz zarur. Shu bilan birga texnik mexanika fanini o'rganish davomida o'quvchilar ishlab chiqarish jaroyonlarini o'rganadilar, ishlab chiqarilgan maxsulotlarni mustahkam, chidamli bo'lishi to'g'risidagi nazariy va amaliy bilimlarga ega bo'lib boradilar.

Agar jismga uzluksiz ravishda kuchlar ta'sir etsa, u harakatlanadi. Ta'sir etuvchi kuchlar davriy takrorlanuvchi bo'lsa u holda jism ham tebranma harakat qiladi. Bunday harakatga majburiy tebranma harakat deyiladi. Jism tebranma harakatini o'rganish tebranma harakat differensial tehglamalarini tuzish va yechimlarini aniqlashga keltiriltiladi. Tebranma harakat differensial tehglamalarini tuzish va yechimlarini aniqlash, ayniqsa uzluksiz muhitlarda joylashgan deformatsiyalanuvchi jismlarni harakat differensial tehglamalarini tuzish va yechimlarini aniqlash izlanuvchidan yuqori matematik mahorat talab qiladi, hamda dolzarb masalalardan hisoblanadi.

ADABIYOTLAR TAHLILI VA METODOLOGIYASI

Elastiklik nazariyasi fani texnik mexanika fanining asoslaridan bo'lib, deformatsiyalanuvchi qattiq jismlarning deformatsiya – kuchlanganlik holatlari o'rganiladi. [1]-Adabiyotda deformatsiya – kuchlanganlik holatlarini o'rganishning potensiallardagi yechish usullari keltiriladi. Ko'p qatlamli muhitlarda to'lqinlarning tarqalish masalalari, ko'chish va kuchlanishlarni hisoblash, chegaraviy shartlarning qo'yilishi keltirilgan.

[2]-Adabiyotda qovushqoq elastik yarim fazoda joylashgan qatlamda tolqinlarning tarqalishi o'rganilgan.

[3, 4]-Adabiyotlarda qobiqlrning umumiy nazariyasi, qobiqlrning texnikada qo'llanilishi, ularni deformatsiya – kuchlanganlik holatlarini hisoblash nazariyalari berilgan.

Shu maqsadda cheksiz elastik muhitda joylashgan elastik silindrik jismlarni davriy kuchlar, bo'ylama va ko'ndalang to'lqinlar *(P, SV, SH)* ta'siridagi majburiy tebranma harakati o'rganildi. Yuqorida keltirilgan adabiyotlardagi manbalardan foydalanildi.

Ta'sir etuvchi to'lqinlar quyidagicha ifodalanadi[2].

$$\varphi^{(p)} = U_0^{(p)} \frac{1}{iK_1} \ell^{i\alpha(x-ct)} \cdot \sum_{n=0}^{\infty} a_{n1} J_n(\gamma r) \cdot \cos(n\theta);$$

$$\psi^{(p)} = U_0^{(p)} \frac{1}{iK_1 \sin\theta_0} \ell^{i\alpha(x-ct)} \cdot \sum_{n=0}^{\infty} b_{n1} J'_{nn}(\delta_1 r) \cdot \sin(n\theta);$$

$$\chi^{(p)} = U_0^{(p)} \frac{1}{iK_2 \cos\theta_0} \ell^{i\alpha(x-ct)} \cdot \sum_{n=0}^{\infty} c_{n1} J_n(\delta_1 r) \cdot \cos(n\theta);$$

(I)

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ISSN(E): 3067-7939
Volume 01, Issue 09, December, 2025
Website: usajournals.org

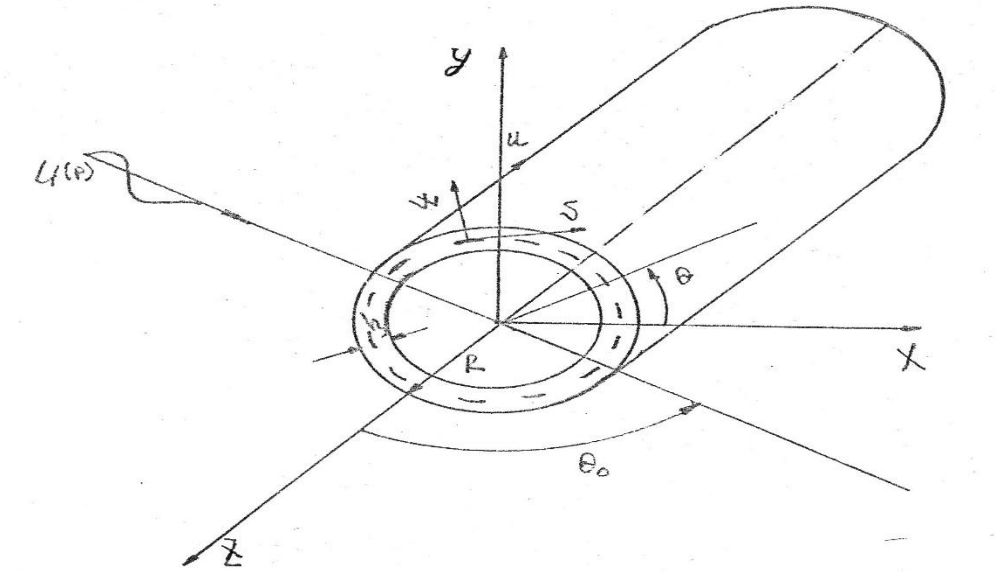
$$\alpha = \begin{cases} K_1 \cos\theta_0; \cdot P\_to'lqinlar\_uchun \\ K_2 \cos\theta_0; \cdot SV, SH\_to'lqinlar\_uchun \end{cases}$$

$$\gamma = \sqrt{K_1^2 + \alpha^2}; \qquad K_1 = \omega / C_{P1}; \quad K_2 = \omega / C_{S1};$$

$$\delta = \sqrt{K_2^2 + \alpha^2};$$

$$\in_n = \begin{pmatrix} a_{n1} \\ b_{n1} \\ c_{n1} \end{pmatrix} = \begin{cases} 1 \cdots n = 0; \\ 2i^n \ldots n \succ 0; \end{cases}$$

To'lqinlar ta'siri 1-shaklda ko'rsatigandek, to'lqin fronti silindrik jism simmetriya o'qiga perpendikulyar bo'ladi.



1-shakl. Hisoblash modeli

Ikkita masalani ko'rib chiqamiz. Silindrik jismlarni silindrik devorlarini qalinligiga ko'ra silindrik qatlam, silindrik qobiq shaklida o'rganamiz.
Birinchi masalada silindrik qatlamlarning turli mexanik sistemalar bilan o'zaro ta'sirdagi ichki kuchlar ta'sirida majburiy tebranishlari qaraladi. $P = P_0 \cdot e^{i\omega t}$;
Silindrik qatlam va mexanik sistema elastik yoki qovushqoq elastik bo'lganda bir jinsli sistema, biri elastik, ikkinchisi qovushqoq elastik bo'lganda bir jinsli bo'lmagan sistema hosil bo'ladi[3].
Elastiklik nazariyasining harakat differentsial tenglamalaridan foydalanamiz[1]:

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$$(\lambda_j + 2\mu_j) graddiv\vec{U}_j + \mu_j rotrot\vec{U}_j = \rho_j \frac{\partial^2 \vec{U}_j}{\partial t^2}; \qquad (1)$$

$\vec{U}_j(r,\theta,z,t) -$ silindrik qatlamlar va tashqi muhitning ko'chish vektori,

$\lambda_j, \mu_j -$ Lyame elastiklik koeffitsiyentlari,

$\rho_j -$ elastik muhit zichligi,

j - silindrik qatlamlar soni.

$\lambda_j, \mu_j -$ Lyame elastiklik koeffitsiyentlari elastiklik moduli va Puasson koeffitsiyenti bilan quyidagicha munosabatda bo'ladi.

$$\lambda_j = \frac{E\nu_j}{(1+\nu_j)(1-2\nu_j)}; \mu_j = \frac{E}{2(1+\nu_j)} \qquad (2)$$

$\vec{U}_j(r,\theta,z,t) -$ ko'chish vektorini Grin – Lyame bo'yicha $\varphi_j, \psi_j -$ potentsiallar orqali ifodalaymiz[2].

$$\vec{U}_j = grad\varphi_j + rot\vec{\psi}_j; \qquad (3)$$

Bo'ylama to'lqinlar uchun $\vec{\psi}_j = 0;$

Yuqoridagilarni hisobga olib (2)ga qo'ysak, u skalyar ko'rinishda quyidagicha yoziladi:

$$\nabla^2\varphi_j - \frac{1}{C_{bj}^2}\frac{\partial^2\varphi_j}{\partial t^2} = 0; \qquad (4)$$

$C_{bj} = \sqrt{\frac{\lambda_j + 2\mu_j}{\rho_j}} -$ bo'ylama to'lqin tezligi,

$\nabla -$ Laplas operatori.

(4) ning elastik muhit va silindrik qatlam uchun yechimlarini

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$$\varphi_1 = AH_0^{(1)}(\alpha_1 r)e^{i\omega t};$$
$$\varphi_2 = [CH_0^{(1)}(\alpha_2 r) + DH_0^{(2)}(\alpha_2 r)]e^{i\omega t}; \quad (5)$$

ko'rinishda olamiz[3]. Bunda, $\alpha_j = \frac{\omega}{p_j};-$ $j = 1,2;$

Elastik muhit va silindrik qatlam chegarasida bikr kontakt sharti qo'yiladi: *r=a;* da $\sigma_{rr_2} = P_0 e^{i\omega t};$ Elastik qatlam ichki chegarasida bo'ylama to'lqinlar ta'sir etadi.

r=b; da $\sigma_{rr_1} = \sigma_{rr_2};$ $U_{rr_1} = U_{rr_2};$ bikr kontaktda elastik muhit va silindrik qatlam chegarasida ko'chish va kuchlanishlar bir xil.

Elastiklik nazariyasidagi ko'chish va kuchlanishning potensiallar bilan bog'lanishlaridan foydalanamiz[2]:

$$U_r = \frac{\partial\varphi}{\partial r} + \frac{1}{r}\frac{\partial\psi}{\partial\theta};$$
$$U_\theta = -\frac{\partial\psi}{\partial r} + \frac{1}{r}\frac{\partial\varphi}{\partial\theta};$$
$$\sigma_{rr} = -2\mu[(a\alpha^2 + D_1)\varphi + D_2\psi];$$
$$\sigma_{r\theta} = 2\mu[-D_2\varphi + \frac{1}{2}(\beta^2 + 2D_1)\psi];$$
$$\sigma_{\theta\theta} = 2\mu[(\frac{\lambda}{2\mu}\alpha^2 + D_1)\varphi + D_2\psi];$$
$$D_1 = \frac{1}{r}(\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial^2}{\partial\theta^2});$$
$$D_2 = \frac{1}{r}\frac{\partial}{\partial\theta}(\frac{1}{r} - \frac{\partial^2}{\partial r^2});$$
$$a = \frac{\lambda + 2\mu}{2\mu}; \quad (6)$$

(5) ni hisobga olib (6) ko'chish va kuchlanishlarni hisoblaymiz:

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$$U_{r1=} = -\frac{A}{r}\alpha_1 r H_1^{(1)}(\alpha_1 r)\cdot e^{i\omega t};$$

$$U_{r2} = -\frac{1}{r}[C\cdot\alpha_2 r\cdot H_1^{(1)}(\alpha_2 r) + D\cdot\alpha_2 r\cdot H_1^{(2)}(\alpha_2 r)]\cdot e^{i\omega t};$$

$$\sigma_{rr1} = \frac{\lambda_1 + 2\mu_1}{r^2} A[-\alpha_1^2 r^2 H_0^{(1)}(\alpha_1 r) + \frac{2\mu_1}{\lambda_1 + 2\mu_1}\alpha_1 r H_1^{(1)}(\alpha_1 r)]\cdot e^{i\omega t};$$

$$\sigma_{rr2} = \frac{\lambda_2 + 2\mu_2}{r^2}\{[\frac{2\mu_2}{\lambda_2 + 2\mu_2}\cdot\alpha_2 r\cdot H_1^{(1)}(\alpha_2 r) - \cdot\alpha_2^2 r^2\cdot H_0^{(1)}(\alpha_2 r)]C + \qquad (7)$$

$$+[\frac{2\mu_2}{\lambda_2 + 2\mu_2}\cdot\alpha_2 r\cdot H_1^{(2)}(\alpha_2 r) - \cdot\alpha_2^2 r^2\cdot H_0^{(2)}(\alpha_2 r)]D\}\cdot e^{i\omega t};$$

(6) va (7) ni chegaraviy shartlarga qo'yib quyidagi kompleks algebraik tenglamalar sistemasini hosil qilamiz[3]:

$$[C]\{g\}=\{P\}; \qquad (8)$$

{g}- noma'lumlar matritsasi;
{P} –tashqi kuchlar matritsasi;
[C] - noma'lumlar oldidagi sonlar matritsasi.

$$\{g\} = \begin{Bmatrix} A \\ C \\ D \end{Bmatrix}; \qquad \{P\} = \begin{Bmatrix} 0 \\ 0 \\ P_0/(\lambda_2 + \mu_2)e^{i\omega t} \end{Bmatrix};$$

[C] - noma'lumlar oldidagi sonlar matritsasi uchinchi tartibli matritsa bo'lib, uning elementlari quyidagicha hisoblanadi:

$$C_{11} = \frac{\lambda_1 + 2\mu_1}{\lambda_2 + 2\mu_2}\left[\frac{2\mu_1}{\lambda_1 + 2\mu_1}\alpha_1 b\cdot H_1^{(1)}(\alpha_1 b) - \cdot\alpha_2^2 b^2\cdot H_0^{(1)}(\alpha_2 b)\right];$$

$$C_{12} = -\frac{2\mu_2}{\lambda_2 + 2\mu_2}\cdot\alpha_2 b\cdot H_1^{(1)}(\alpha_2 b) + \alpha_2^2 b^2\cdot H_0^{(1)}(\alpha_2 b);$$

$$C_{13} = -\frac{2\mu_2}{\lambda_2 + 2\mu_2}\cdot\alpha_2 b\cdot H_1^{(2)}(\alpha_2 b) + \alpha_2^2 b^2\cdot H_0^{(1)}(\alpha_2 b);$$

$$C_{21} = \alpha_1 b\cdot H_1^{(1)}(\alpha_2 b); \qquad (9)$$

$$C_{22} = -\alpha_2 b\cdot H_1^{(1)}(\alpha_2 b);$$

$$C_{23} = -\alpha_2 b\cdot H_1^{(2)}(\alpha_2 b);$$

$$C_{31} = 0;$$

$$C_{32} = \left[\frac{2\mu_2}{\lambda_1 + 2\mu_1}\alpha_2 a\cdot H_1^{(1)}(\alpha_2 a) - \cdot\alpha_2^2 a^2\cdot H_0^{(1)}(\alpha_2 a)\right]/a^2;$$

$$C_{33} = \left[\frac{2\mu_2}{\lambda_1 + 2\mu_1}\alpha_2 a\cdot H_1^{(2)}(\alpha_2 a) - \cdot\alpha_2^2 a^2\cdot H_0^{(2)}(\alpha_2 a)\right]/a^2;$$

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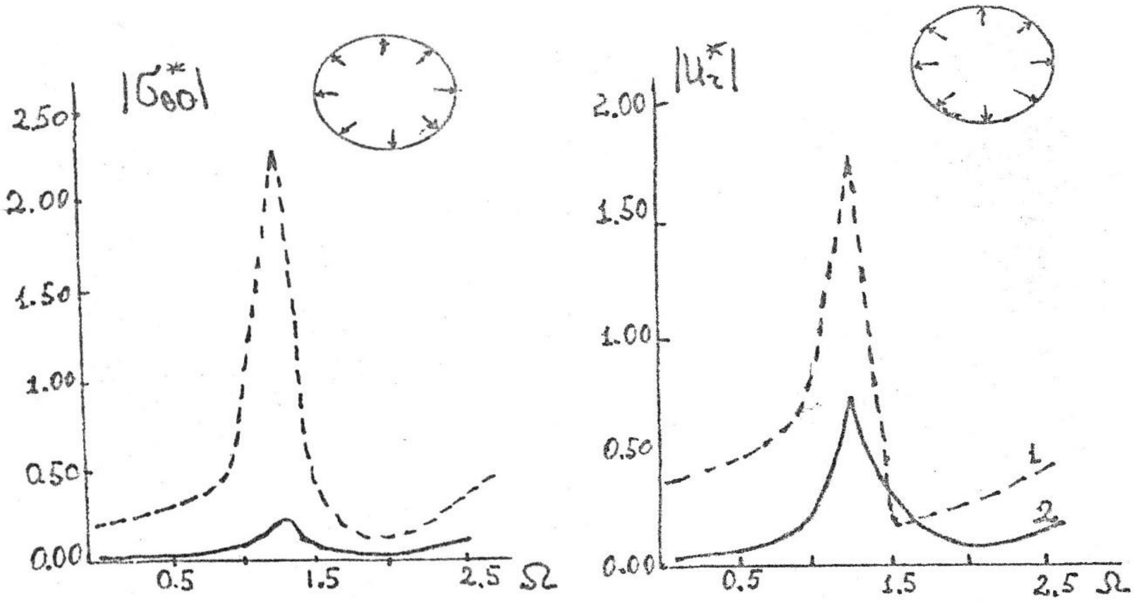
Hosil bo'lgan (8) algebraik tenglamalar sistemasi (9) ni e'tiborga olgan holda, Gaus usulida a=2; b=2,2 qiymatlarda yechildi. U_{rr} −radial ko'chish, $\sigma_{\theta\theta}$ −xalqa ko'ndalang kesimiga normal(ning) kuchlanishlarining to'lqin soniga nisbatan o'zgarishi o'rganildi. Elastik muhitda joylashgan silindrik qatlamda bo'ylama to'lqin ta'sirida $\sigma_{\theta\theta}$ - doiraviy kuchlanish maksimal qiymatga erishadi va so'nadi, cheksiz elastik muhit tebranishlarni so'ndiradi.



2-shakl. Doiraviy kuchlanish va radial ko'chishlar to'lqin soniga nisbatan o'zgarishi.

Ikkinchi masalada cheksiz muhitda joylashgan silindrik qobiqni garmonik to'lqinlar ta'siridagi dinamik o'zaro ta'sirlarini o'rganamiz. Seysmik to'lqinlar chastotasi ω, amplitudasi U_0.

Majburiy tebranma harakatda elastik muhit va silindrik qatlam uchun (4) xususiy hosilali differensial tenglama yechimlarini (5) yechimlar kabi qidiramiz. Chekiz muhitda Zommerfeld sharti inobatga olinadi. (I) dan hosil bo'lgan yechimlar bilan (5) yechimlar yig'indisi yechimlari – to'liq yechimlari ifodalaydi.

Elastik muhit uchun ko'chish va kuchlanishlar ifodasi quyidagicha bo'ladi:

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$$U_{r1}=\sum_{n=0}^{\infty}\left[A_{n1}\gamma\cdot K_n'(\gamma r)+B_{n1}K_n(\delta r)\frac{n}{r}+\delta(i\alpha)C_{n1}K_n'(\delta r)\right]X_1\ell^{i\alpha(x-ct)};$$
$$U_{\theta 1}=\sum_{n=0}^{\infty}\left[A_{n1}\cdot K_n(\gamma r)\frac{n}{r}+B_{n1}\delta\cdot K_n'(\delta r)+\frac{n}{r}(i\alpha)C_{n1}K_n(\delta r)\right]X_2\ell^{i\alpha(x-ct)};$$
$$U_{x1}=\sum_{n=0}^{\infty}\left[A_{n1}i\alpha\cdot K_n'(\gamma r)+B_{n1}K_n(\gamma r)-C_{n1}\delta(\frac{1}{r}K_n'(\delta r)+\delta K_n''\ (\delta\alpha))-\frac{n}{r^2}K_n(\delta r)\right]X_1\ell^{i\alpha(x-ct)};$$

$$\sigma_{rr1}=\sum\left\{A_{n1}\left[\lambda_r\left(\frac{1}{r}K_n'(\gamma r)+(\frac{n^2}{r^2}-\alpha^2)K_n(\gamma r)+2\mu_nK_n''(\gamma r)\right)\right]+B_n\left[\lambda_n\tfrac{n}{r}(1-\frac{1}{r})K_n(\delta r)+2\mu_n\frac{n}{r}K_n'(\delta r)\right]+\right.$$
$$\left.+C_{n1}\left[(\lambda_n+2\mu_n)i\alpha\ell K_n'(\delta r)-i\alpha(\frac{n^2}{r^2}\alpha+\delta^2)K_n(\delta r)\right]\right\}X_1\ell^{i\alpha(x-ct)};$$
$$\sigma_{r\theta 1}=\sum_{n=0}^{\infty}\left\{\frac{n}{r}\mu_nA_{n1}\left[\frac{1}{r}K_n(\gamma r)-K_n'(\gamma r)\right]+\frac{\mu_n}{2}B_{n1}\left[\frac{1}{r}K_n'(\gamma r-\frac{n^2}{r^2}K_n(\delta r)-K_n''(\delta r)\right]+\right.$$
$$\left.+\frac{n\mu_n}{2r}C_{n1}\left[i\alpha\ell\frac{2}{r}K_n(\delta r)-K_n'(\delta r)\right]\right\}X_2\ell^{i\alpha(x-ct)};$$
(10)
$$\sigma_{rx1}=\sum_{n=0}^{\infty}\left[\mu_n\delta rA_{n1}+B_{n1}\frac{i\alpha}{2r}\mu_nK_n(\delta r)+C_{n1}i(\alpha\ell-\delta^2)K_n'(\delta r)\right]X_1\ell^{i\alpha(x-ct)};$$

Silindrik qobiqga *P, SH, SV* garmonik to'lqinlardan biri, masalan, *P* bo'ylama to'lqin ta'sir etsin.

Undan qobiqni o'rab turuvchi muhit kuchlanishlarini (6) formulalar yordamida hisoblaymiz:

$$\sigma_{rr}^{(p)}=U_0^{(p)}\frac{1}{iK_1}\ell^{i\alpha(x-ct)}\cdot\sum_{n=0}^{\infty}\in_n\left[\lambda_n\left(\frac{1}{r}J_n'(\gamma r)+J_n''(\gamma r)-(\frac{n^2}{r^2}+a^2)J_n(\gamma r)\right)+2\mu_nJ_n''(\gamma r)\right]\cos n\theta;$$
$$\sigma_{r\theta}^{(p)}=-2\mu_n\ell^{i\alpha(x-ct)}\frac{U_0^{(p)}}{iK_1}(i\alpha-\frac{1}{r})\sum_{n=0}^{\infty}\in_n nJ_n(\gamma r)\sin(n\theta);$$
$$\sigma_{rx}^{(p)}=-2\mu_n\ell^{i\alpha(x-ct)}\cdot\frac{U_0^{(p)}}{iK_1}(i\alpha)\sum_{n=0}^{\infty}\in_n nJ_n(\gamma r)\sin(n\theta);\qquad(11)$$

Quyidagicha belgilash kiritamiz.

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$$S_{1n} = \in_n \left[\lambda_n \left(\frac{1}{r} J_n'(\gamma r) + J_n''(\gamma r) - (\frac{n^2}{r^2} + a^2) J_n(\gamma r) \right) + 2\mu_n J_n''(\gamma r) \right];$$

$$S_{2n} = 2\mu_n (i\alpha - \frac{1}{r}) \in_n n \cdot J_n(\gamma r);$$

$$S_{3n} = 2\mu_n (i\alpha) \in_n n \cdot J_n(\gamma r); \qquad (12)$$

Natijalovchi kuchlanish va ko'chishlar ikkita natijalovchi kuchlanish va ko'chishlar yig'indisidan tashkil topadi.

$$U_r = U_{r1} + U_r^{(p)}; \qquad U_\theta = U_{\theta 1} + U_\theta^{(p)}; \qquad U_x = U_{x1} + U_x^{(p)};$$

$$\sigma_{rr} = \sigma_{rr1} + \sigma_{rr}^{(p)} \qquad \sigma_{\theta r} = \sigma_{r\theta 1} + \sigma_{\theta r}^{(p)} \qquad \sigma_{rx} = U_{rx1} + U_{rx}^{(p)} \qquad (13)$$

$$\sigma_{\theta\theta} = \sigma_{\theta\theta 1} + \sigma_{\theta\theta}^{(p)} \qquad \sigma_{xx} = \sigma_{xx1} + \sigma_{xx}^{(p)} \qquad \sigma_{x\theta} = \sigma_{x\theta 1} + \sigma_{x\theta}^{(p)}$$

(12), (10) larni hisobga olib natijalovchi kuchlanish va ko'chishlarni quyidagicha yoza olamiz:

$$\sigma_{rr} = \sum_{n=0}^{\infty} \left\{ U_0^{(p)} \frac{1}{iK_1} S_{1n} \cos(n\theta) + A_{n1} \left[\lambda_r \left(\frac{1}{r} K_n'(\gamma r) + (\frac{n^2}{r^2} - \alpha^2) K_n(\gamma r) + 2\mu_n K_n''(\gamma r) \right) \right] + \right.$$

$$\left. + B_n \left[\lambda_n \tfrac{n}{r} (1 - \frac{1}{r}) K_n(\delta r) + 2\mu_n \frac{n}{r} K_n'(\delta r) \right] + C_{n1} \left[(\lambda_n + 2\mu_n) i\alpha\ell K_n'(\delta r) - i\alpha(\frac{n^2}{r^2}\alpha + \delta^2) K_n(\delta r) \right] \right\} X_1 \ell^{i\alpha(x-ct)};$$

$$\sigma_{r\theta} = \sum_{n=0}^{\infty} \left\{ U_0^{(p)} \frac{1}{iK_1} S_{2n} \sin(n\theta) + \frac{n}{r} \mu_n A_{n1} \left[\frac{1}{r} K_n(\gamma r) - K_n'(\gamma r) \right] + \frac{\mu_n}{2} B_{n1} \left[\frac{1}{r} K_n'(\gamma r - \frac{n^2}{r^2} K_n(\delta r) - K_n''(\delta r) \right] + \right.$$

$$\left. + \frac{n\mu_n}{2r} C_{n1} \left[i\alpha\ell \frac{2}{r} K_n(\delta r) - K_n'(\delta r) \right] \right\} X_2 \ell^{i\alpha(x-ct)};$$

$$\sigma_{rx} = \sum_{n=0}^{\infty} \left[U_0^{(p)} \frac{1}{iK_1} S_{3n} \sin(n\theta) + \mu_n \delta r A_{n1} + B_{n1} \frac{i\alpha}{2r} \mu_n K_n(\delta r) + C_{n1} i(\alpha\ell - \delta^2) K_n'(\delta r) \right] X_1 \ell^{i\alpha(x-ct)};$$

$$U_r = \sum_{n=0}^{\infty} \left[U_0^{(p)} \frac{1}{iK_1} \in_n J_n(\gamma r) \cos(n\theta) + [A_{n1}\gamma \cdot K_n'(\gamma r) + B_{n1} K_n(\delta r) \frac{n}{r} + \delta(i\alpha) C_{n1} K_n'(\delta r)] X_1 \right] \ell^{i\alpha(x-ct)};$$

$$U_\theta = \sum_{n=0}^{\infty} \left[\frac{1}{r} U_O^{(p)} \frac{\in_n}{iK_1} n J_n(\gamma r) \sin n\theta + [A_{n1} \cdot K_n(\gamma r) \frac{n}{r} + B_{n1}\delta \cdot K_n'(\delta r) + \frac{n}{r}(i\alpha) C_{n1} K_n(\delta r)] X_2 \right] \ell^{i\alpha(x-ct)};$$

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$$U_X=\sum_{n=0}^{\infty}\left[\frac{1}{r}U_O^{(p)}\frac{\in_n}{iK_1}nJ_n(\gamma r)\cdot\cos(n\theta)+[A_{n1}(ai)K_n(\gamma r)-C_{n1}e\left(\frac{1}{r}K_n{}'(\delta r)\right)+K_n^{"}(\delta\alpha)-\frac{n}{r^2}K_n(\delta r)]X_1\right]\cdot\ell^{ia(x-ct)};$$

(14)

$$\begin{pmatrix}U_r^{(p)}\\U_\theta^{(p)}\\U_x^{(P)}\end{pmatrix}=U_O^{(p)}\begin{pmatrix}1\\-r^{-1}\\ia\end{pmatrix}\frac{1}{iK_1}e^{ia(x-ct)}\times\sum_{n=o}^{\infty}\begin{pmatrix}J_n^1(\gamma r)\\nJ_n(\gamma r)\\J_n(\gamma r)\end{pmatrix}\begin{pmatrix}X_1\\X_2\\X_1\end{pmatrix}\quad(15)$$

Elastik muhit va silindrik jism chegarasida ko'chishlarning uzluksizligi sharti qo'yiladi.

$$U_{r1}=W-U_r^{(p)}\qquad U_{\theta 1}=V-U_\theta^{(p)}\qquad U_{x1}=U-U_x^{(p)}\qquad(16)$$

(16) chegaraviy shartlardan An1,Bn1, Cn1 doimiylar aniqlanadi. Silindrik qobiqlarning U,V,W yechimlari matritsa korinishda olinadi[4].

$$U=\sum_{n=o}^{\infty}U_n^{(1)}X_1e^{ia(x-ct)}$$

$$V=\sum_{n=o}^{\infty}V_n^{(1)}X_2e^{ia(x-ct)}\qquad(17)$$

$$W=\sum_{n=o}^{\infty}W_n^{(1)}X_1e^{ia(x-ct)}$$

$U_n^{(1)},V_n^{(1)},W_n^{(1)}$ –doimiylar (16) chegaraviy shartlardan aniqlanadi. Hisoblangan ko'chishlar, kuchlanishlar va silindrik qobiqlarning

$$\frac{\partial^2 V}{\partial t^2}+\frac{\partial W}{\partial\theta}+\frac{h^2}{12R^2}\left(\frac{\partial^2 V}{\partial\theta^2}-\frac{\partial^3 W}{\partial\theta^3}\right)=\frac{R^2}{c^2}\left(\frac{\partial^2 V}{\partial t^2}-\frac{1}{\rho_0 h}\sigma_{r\theta n/R}\right)$$

$$\frac{\partial V}{\partial\theta}+W+\frac{h^2}{12R^2}\left(\frac{\partial^4 V}{\partial\theta^4}-\frac{\partial^3 V}{\partial\theta^3}\right)=\frac{R^2}{c^2}\left(\frac{\partial^2 W}{\partial t^2}-\frac{1}{\rho_0 h}\sigma_{rrn/R}\right)\qquad(18)$$

differensial tenglamani qanoatlantiuvchi U,V,W yechimlarini (13) chegaraviy shartlarga qo'yib, quyidagi algebraik tenglamalar sistemasini hosil qilamiz.

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$$1.\sum_{n=0}^{\infty}\left[A_{n1}K_n^{'}(\gamma R)+B_nK_n^{'}(\delta R)\frac{n}{R}+e(ia)C_{n1}K_n(\delta R)\right]X_1e^{ia(x-ct)}+$$

$$+U_o^{(p)}\frac{1}{iK_1}e^{ia(x-ct)}=\sum_{n=o}^{\infty}W_n^{(1)}X_1e^{ia(x-ct)}$$

$$2.\sum_{n=o}^{\infty}\left[-\frac{n}{R}A_{n1}K_n^{'}(\gamma R)-B_{n1}K_n^{'}(\delta R)-\frac{\delta}{R}(ia)C_{n1}K_n(\delta R)\right]X_2e^{ia(x-ct)}-$$

$$-\frac{1}{R}U_O^{(p)}\frac{1}{iK_1}e^{ia(x-ct)}\sum_{n=o}^{\infty}\in_n nJ_n(\gamma R)\sin n\theta=$$

$$=\sum_{n=o}^{\infty}V_n^{(1)}X_2e^{ia(x-ct)}$$

$$3.\sum_{n=o}^{\infty}\left[A_{n1}(ai)K_n(\gamma R)-C_{n1}e\left(\frac{1}{r}K_n^{'}(\delta R)+K_n^{''}(\delta R)-\frac{n^2}{r^2}K_n(\delta R)\right)\right]X_1e^{ia(x-ct)}=$$

$$=\sum_{n=o}^{\infty}U_n^{(1)}X_1e^{ia(x-ct)}$$

$$4.\sum_{n=o}^{\infty}\left[-\alpha^2U_n^1-\frac{1-v_o}{2R^2}V_n^1+\frac{1+v_0}{2R}ianV_n^{(1)}+\frac{v_o}{R}iaW_n^{(1)}+\frac{1-v_o^2}{E_oh_o}\rho_o w^2U_n^{(1)}\right]X_1e^{ia(x-ct)}+$$

$$+\sum_{n=o}^{\infty}\left[\mu_rA_{n1}(ai)K_n(\gamma R)+B_{n1}\frac{ia}{2R}\mu_rK_n(\delta R)+C_{n1}\mu_ri\left(ae^2-\delta^2\right)\right]K_n^{'}(\delta R)X_1e^{ia(x-ct)}=$$

$$=\frac{1-v_0^2}{E_oh_o}U_O^{(p)}\frac{1}{iK_1}e^{ia(x-ct)}\sum_{n=o}^{\infty}S1_n\cos n\theta$$

$$5.\sum_{n=o}^{\infty}\left\{\frac{1+v_o}{2R}(ian)U_n^{(1)}+\left[\frac{1+v_o}{2}(ia)^2-\frac{n^2}{R^2}+\frac{1-v_o^2}{E_oh_o}\rho_O w^2\right]V_n^{(1)}+\frac{\alpha}{R^2}W_n^{(1)}+\frac{1-v_o^2}{E_oh_o}\times\right.$$

$$+\left(\frac{n}{R}\mu_rA_{n1}\left(\frac{1}{R}K_n(\gamma R)-K_n^{'}(\gamma R)\right)+\frac{\mu_r}{2}B_{n1}\left(\frac{1}{R}K_n^{'}(\delta R)-\frac{n}{R^2}K_n(\delta R)\right)-K_n^{''}(\delta R)\right)+$$

$$+\frac{n\mu_r}{2R}C_{n1}(iae)\left(\frac{2}{R}K_n(\delta R)-K_n^{'}(\delta R)\right)X_2e^{ia(x-ct)}=\frac{1-v_o^2}{E_oh_o}U_o^{(p)}\frac{1}{iK_1}e^{ia(x-ct)}\sum_{n=o}^{\infty}S2_n\sin n\theta$$

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$$6.\sum_{n=o}^{\infty}\left\{ia\frac{v_o}{R}V_n^{(1)}+\frac{n}{R^2}V_n^{(1)}+\left[\frac{h^2}{12}\left(\alpha^4+\frac{2}{R^2}+\frac{n}{R^4}\right)-\frac{1-v_0^2}{E_oh_o}\rho_o w^2\right]W_n^{(1)}+\right.$$

$$+\frac{1-v_o^2}{E_oh_o}\left(A_{n1}\left(\lambda_r\left[\frac{1}{R}K_n^{'}(\gamma R)+\frac{n}{R^2}^{2}K_n(\gamma R)-\alpha^2K_n(\gamma R)\right]+2\mu_rK_n^{''}(\gamma R)\right)\right)+$$

$$+B_{n1}\left(\lambda_r\frac{n}{R}\left(1-\frac{1}{R}\right)K_n^{'}(\delta R)+2\mu_r\frac{n}{R}K_n^{'}(\delta R)\right)+C_{n1}\left[\lambda_r(iae)K_n^{''}(\delta R)\right]+$$

$$+ia\left(\frac{n^2}{R^2}\alpha+\delta^2\right)K_n(\delta R)+2\mu_rieaK_n(\delta R)X_1e^{ia(x-ct)}=$$

$$=-\frac{1-v_o^2}{E_oh_o}U_o^{(p)}\frac{1}{iK_1}e^{ia(x-ct)}\sum_{n=o}^{\infty}S1_n\sin n\theta$$

Bu tenglamalar sistemasidan A_{n1}, B_{n1}, C_{n1} noma'lumlarni aniqlaymiz.

Silindrik qobiq xalqa va o'q bo'ylab yo'nalgan $S_{\theta\theta}, S_{xx}$ zo'riqishlarini hisoblaymiz va taqqoslaymiz.

$$S_{xx}=\left[ci\alpha U+\frac{Cv_o}{R}\left(W+\frac{\partial V}{\partial\theta}\right)\right]X_1e^{ia(x-ct)}$$

$$S_{\theta\theta}=\left[ci\alpha\cdot v_o\cdot U+\frac{C}{R}\left(W+\frac{\partial V}{\partial\theta}\right)\right]X_2\ell^{ia(x-ct)};$$

Bunda,

$$C=\frac{E_0h_0}{(1-v_0^2)};$$

NATIJA VA MUHOKAMA

Aniqlangan zo'riqishlarni

$$\sigma_0^{(p)}=\mu_rU_o^{(p)}K_1\frac{C_{p_1}^2}{C_{s_1}^2};$$

ga bo'lib, o'lchamsiz holga keltiramiz. O'lchamsiz xalqaviy zo'riqish θ o'sishi bilan o'sadi, shu vaqtda o'q bo'ylab yo'nalgan zo'riqish kamayadi. Xalqaviy kuchlanish $\theta = 90^{\circ}$ da maksimal qiymatga erishadi.

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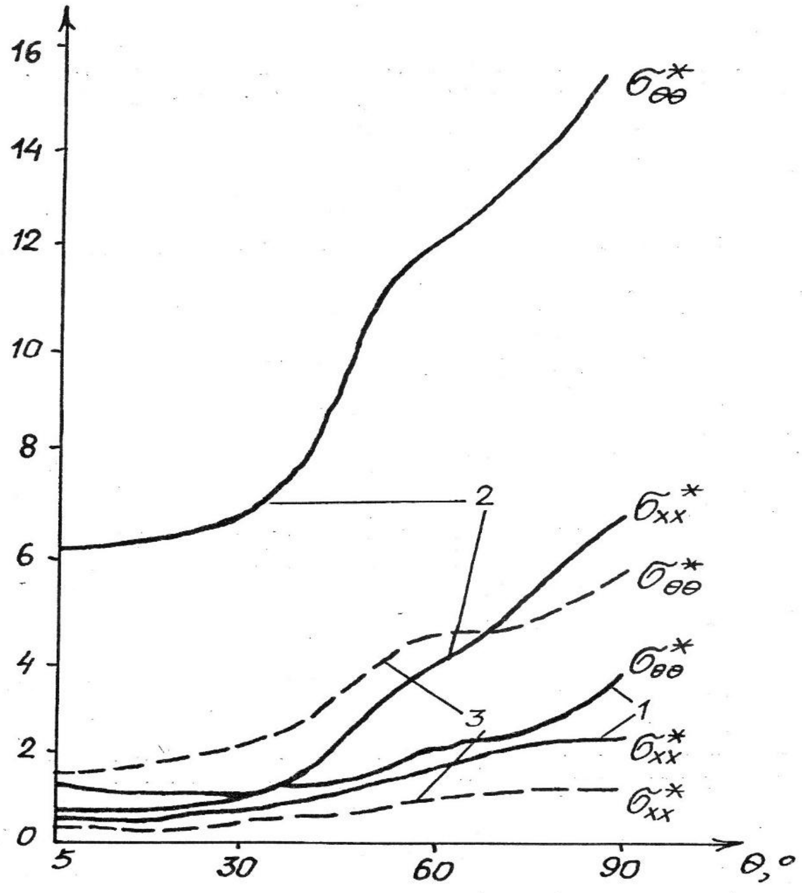

3-shakl. Kuchlanganlik holatining tushish burchagiga nisbatan o’zgarishi

XULOSA

Ikkita masala yechildi. Birinchi masalada ichki garmonik kuchlar ta’siridagi silindrik qatlmning bo’ylama to’lqin ta’sirida doiraviy kuchlanish o’rganildi. $\sigma_{\theta\theta}$ - doiraviy kuchlanish va radial ko’chishlar dastlab maksimal qiymatga erishadi va so’nadi, cheksiz elastik muhit tebranishlarni so’ndiradi.

Ikkinchi masalada cheksiz muhitda joylashgan silindrik qobiqni garmonik to’lqinlar ta’siridagi dinamik o’zaro ta’sirlarini o’rganildi. $\sigma_{\theta\theta}$ Xalqaviy va σ_{xx} o’q bo’ylab yo’nalgan kuchlanish θ o’sishi bilan o’sadi, shu vaqtda o’q bo’ylab yo’nalgan zo’riqish kamayadi. Xalqaviy kuchlanish $\theta = 90^{\circ}$ da maksimal qiymatga erishadi.

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