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# METHODOLOGY FOR SIMPLIFYING THE ZHEGALKIN POLYNOMIAL IN BOOLEAN ALGEBRA

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## Abstract

This article discusses the mathematical essence of the Zhegalkin polynomial in logical algebra and the methodology for simplifying expressions through specific examples in the process of teaching the subject "Discrete Structures" to students of the "Artificial Intelligence" educational program in technical higher education institutions.

**Keywords:** Higher education, artificial intelligence, discrete structure, algebra of logic, logical functions, Zhegalkin multiplication, logical formula, conjunction, symmetric division, simplification, algorithm.

## Introduction

Boolean algebra is one of the fundamental branches of discrete mathematics and is widely applied in the fields of algorithm theory, digital circuits, cryptography, and artificial intelligence. The problem of representing logical functions in various forms and simplifying them has significant theoretical and practical importance in these areas.



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The decree of the President of the Respublic of Uzbekistan Shavkat Mirziyoyev dated June 5, 2018, No. PQ-3775. “On additional measures to improve the quality of education in higher education institutions and ensure their active participation in the comprehensive reforms being implemented in the country”; the Decree dated May 7, 2020, No. PQ-4708, “On measures to improve the quality of education in the field of mathematics and develop scientific research”; and the Decree dated Febuary 17, 2021, No. PQ-4996. “On measures to create conditions for the accelerated development of artificial intelligence technologies”, as well as other regulatory legal documents, define a number of tasks. With the aim of contributing, at least in part, to the implementation of these tasks, this article has been prepared [1,2,3].

In technical higher education institutions, during the teaching of the course “Discrete Structures” to students of the “Artificial Intelligence” educational pogram, the Zhegalkin polynomial in boolean algebra serves as one of the most effective methods for representing logical functions, as it allows function values to be expressed in an algebraic form. However, in practical problems, the need arises to reduce the number of terms in the polynomial and bring it to a simpler form.

This article analyzes the Zhegalkin polynomial, which is widely used in Boolean algebra when teaching the course “Discrete Structures” to students of the “Artificial intelligence” educational program in technical higher education institutions, as well as its mathematical foundations and methods for simplifying expressions. In addition, the methodology for simplifying The Zhegalkin polynomial is scientifically substantiated, and the possibilities of its application in the educational process are highlighted.

### **Literature Review**

The theory of the Zhegalkin polynomial is associated with the name of the Russian mathematician I.I.Zhegalkin, who proposed the idea of expressing logical functions using algebraic expressions. This approach was later widely developed in Boolean algebra and digital circuit theory.

In K. Rosen’s work “Discrete Mathematics and Its Applications”, the Zhegalkin polynomial is considered as one of the basic forms of repressing logical funtions.



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The author explains the uniqueness of the Zhegalkin polynomial and algorithms for its construction through illustrative examples.

In the works of E. Mendelson and R. Grimaldi, the Zhegalkin polynomial is analyzed in connection with the optimization of logical circuits. These sources provide a scientific justification for the efficiency of circuits based on logical operations.

In the textbook “Discrete Structures” published in 2025 and authored by the present author, theoretical and practical information on Zhegalkin polynomials and their analysis in Boolean algebra is provided.

In recent years, special attention has been paid in published scientific articles and literature to the issues of simplifying logical functions using computer technologies, algebraic systems, and software tools. This enables integration with modern information technologies.

### **Methodological Approach**

When simplifying Zhegalkin polynomials, the following issues should be taken into account:

- ✓ The size of given expression (polynomial) should not be excessively large;
- ✓ The redundant logical elements in practical circuits (truth tables) should not increase.
- ✓ The processes of simplification and analysis should not become overly complicated.

The algebraic simplification of Zhegalkin polynomials is carried out based on the fundamental laws and properties of Boolean algebra. This approach is convenient for expressions of small size and is suitable for manual calculations.

In the process of simplifying expressions, logical functions are analyzed based on truth tables, converted into Zhegalkin polynomials, and algebraic simplification operations are applied, including the identification and elimination of redundant terms.

### **Method, Results, and Discussion.**

**Basic concepts.** To facilitate the analysis of all considerations, expressions can be transformed into a common standard form using logical laws.



**Definition 1.** A logical formula expressed using conjunction and symmetric difference operations is called a Zhegalkin polynomial [11].

1) Transform the given formula into Disjunctive Normal Form (DNF);

2) Eliminate the disjunction operation using the following formula;

$$x \vee y = \overline{x} \wedge \overline{y}$$

3) Replace the negation operation with the symmetric difference operation;

$$\overline{x} = x \otimes 1$$

4) Simplify the resulting expression using the corresponding equality.

$$x \otimes x = 0$$

**Example 1.**  $x \rightarrow y = \overline{x} \vee y$  let us convert the expression into a Zhegalkin polynomial [11].

*Solution.* We apply the steps described above:

$$\begin{aligned} x \rightarrow y &= \overline{x} \vee y = \overline{\overline{x}} \wedge \overline{\overline{y}} = \overline{x} \wedge \overline{y} = (x \wedge (y \otimes 1)) \otimes 1 = \\ &= (x \wedge y \otimes x) \otimes 1 = x \wedge y \otimes x \otimes 1. \end{aligned}$$

**Definition 2.** A conjunction that does not contain negated variables is called a *monotone conjunction* [11].

The number of variables combined by the conjunction operation is called the *degree of the polynomial*.

**Definition 3.** The highest degree among the terms included in a polynomial is called the *degree of the Zhegalkin polynomial* [11].

**Example 2.**  $\alpha(A, B) = A \wedge B \rightarrow (\overline{A \vee B})$  let us convert the formula into a Zhegalkin polynomial [11].

*Solution.*  $x \rightarrow y = \overline{x} \vee y$ . Using the formula, we expand the implication. In this case  $x = A \wedge B$ ,  $y = (\overline{A \vee B})$  it follows that

$$\alpha(A, B) = (\overline{A \wedge B}) \vee (\overline{A \vee B}).$$

$$(\overline{A \wedge B}) = \overline{A} \vee \overline{B} \text{ via } (\overline{A \wedge B}) = \overline{A} \wedge \overline{B} - \text{Applying De Morgan's laws}$$

$$\alpha(A, B) = (\overline{A} \vee \overline{B}) \vee (\overline{A} \wedge \overline{B})$$

And we obtain the expression in the form of.

According to the Absorption Law:

$$(\overline{A} \vee \overline{B}) \vee (\overline{A} \wedge \overline{B}) = \overline{A} \vee \overline{B}$$



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Now, we transform it into a Zhegalking polynomial.

$$\bar{A} = 1 \otimes A, \quad \bar{B} = 1 \otimes B, \quad A \vee B = A \otimes B \otimes A \wedge B$$

By utilizing the following formulas

$$\bar{A} \vee \bar{B} = (1 \otimes A) \otimes (1 \otimes B) \otimes (1 \otimes A) \wedge (1 \otimes B)$$

the expression takes the form.

Based on the above, we perform the calculation:

$$(1 \otimes A) \wedge (1 \otimes B) = 1 \otimes A \otimes B \otimes A \wedge B.$$

Summing all terms:

$$(1 \otimes A) \otimes (1 \otimes B) \otimes (1 \otimes A \otimes B \otimes A \wedge B)$$

By canceling out identical terms

$$1 \otimes A \wedge B$$

we arrive at.

$$\alpha(A, B) = 1 \otimes A \wedge B$$

Thus, we obtain the Zhegalkin polynomial in the following form[11].

The results demonstrate that the following steps are effective in simplifying a Zhegalkin polynomial:

- ✓ Writing all terms in standard order;
- ✓ Simplifying identical terms ( $x \oplus x = 0$ );
- ✓ Identifying high-degree terms that do not affect the function's value;
- ✓ Retaining the minimal set of terms.

Consequently, the simplified Zhegalkin polynomial is com.

## Conclusion

In Boolean algebra, the Zhegalkin polynomial is considered a convenient and unique form of representing logical functions. The methodology of bringing it to its simplest form serves to optimize logical functions, reduce computational complexity, and foster the development of student's logical thinking during the educational process.

## Proposal

During the teaching of the "Discrete Structures" subject to "Artificial Intelligence" students at technical higher education institutions, the effective use of information technologies, algebraic systems, and programming languages in simplifying the Zhegalkin polynomial will enhance educational efficiency..



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