



THE MATHEMATICAL FOUNDATIONS AND SOCIAL SIGNIFICANCE OF INHERITANCE ISSUES IN THE WORKS OF MUHAMMAD IBN MUSA AL-KHWARIZMI

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Abstract

The article examines the third section of Al-jabr wa al-muqabala, entitled “The Book of Wills,” and analyzes its social and philosophical significance. It argues that Muhammad ibn Musa al-Khwarizmi demonstrates the interconnection between algebraic methods and Islamic legal principles, highlighting the role of mathematical modeling in rationalizing social relations. The study further substantiates that al-Khwarizmi should be regarded not only as the founder of algebra, but also as one of the earliest thinkers to integrate legal justice with mathematical reasoning.

Keywords: Muhammad ibn Musa al-Khwarizmi, Algebra, “The Book of Wills,” Inheritance Law, Mathematical Modeling, Social Rationality, Linear Equations, Probability Theory.

Introduction

In the history of world science, the names of certain scholars are inscribed in enduring prominence. Among them is Muhammad ibn Musa al-Khwarizmi, who stands as one of the great intellectual figures of the early Renaissance tradition. Through his works, he made substantial contributions to the development of political, socio-historical, scientific-philosophical, ethnogenetic, and linguistic thought.



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Al-Khwarizmi occupies a distinctive place in the history of mathematics, astronomy, and geography. His writings exerted a significant and lasting influence on the advancement of progressive thought in both the East and the West. He lived and worked during a period when the cities of Central Asia were emerging as major cultural centers following their gradual liberation from the dominance of the Arab Caliphate. At that time, Arabic functioned as the principal language of science, culture, religion, and governance. Having mastered this language, al-Khwarizmi composed all of his works in Arabic, a factor that contributed greatly to his global recognition. It is noteworthy that he was among the first Central Asian scholars to produce scientific works in Arabic.

Al-Khwarizmi's treatises on arithmetic and Algebra initiated a new phase in the history of mathematics, marking the beginning of medieval mathematical science and profoundly shaping its subsequent development. His works served as foundational sources for numerous later studies; they were widely commented upon, and many of their elements were incorporated into subsequent scholarly writings. In his mathematical texts, the scholar addressed practical social issues, compiling knowledge that was both essential for specialists and accessible to the broader population, presenting it concisely yet with conceptual clarity.

Research in the history of mathematics, particularly when approached from a dialectical-materialist perspective, constitutes an important component of contemporary philosophical inquiry. By elucidating the evolution of natural-scientific ideas alongside social thought, such studies contribute to a deeper understanding of the dialectics of scientific methodology and the formation of diverse philosophical frameworks.

This article, drawing on the principles of historical-materialist interpretation, examines the genesis of social problems as reflected in the treatise "The Book of Wills." It seeks to demonstrate how al-Khwarizmi's mathematical reasoning engages with issues of inheritance, Islamic legal norms, and human agency, thereby revealing the integration of abstract mathematical thought with the realities of everyday social life.

The third part of Muhammad ibn Musa al-Khwarizmi's algebraic treatise constitutes approximately half of the entire work and is entitled "The Book of Wills." This section sets out methods for resolving issues related to the



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distribution of inheritance in accordance with Islamic legal principles. The present study relies on the Uzbek translation of this text.

The Arabic manuscript of *Kitab al-Jabr wa al-Muqabala* comprises 34 folios and is structured into three main parts: (1) an algebraic section (folios 1b–15a-b), concluding with a short subsection on commercial transactions; (2) a geometrical section (folios 15a–18b), which applies algebraic methods to measurement problems; and (3) a section devoted to wills (folios 18b–34a-b), which al-Khwarizmi himself designated as “The Book of Wills.”

During al-Khwarizmi’s period, considerable attention was devoted to formulating rules for solving such legal problems through algebraic methods. In many later Arabic mathematical treatises, the theoretical component of algebra was often regarded merely as an introduction to the “science of inheritance distribution” (*‘ilm al-farā’id*). A notable example is provided by the works of the twelfth-century scholar Siraj al-Din al-Sajawandi, whose treatises were widely disseminated in Central Asia.

It is significant that “The Book of Wills” is absent from all known Latin translations and adaptations of al-Khwarizmi’s work. This fact was first noted in 1911 by Louis Charles Karpinski. Evidently, medieval European mathematicians showed limited interest in issues pertaining to Islamic jurisprudence.

For a long time, historians of mathematics also paid little attention to this section of al-Khwarizmi’s *Algebra*. Franz Rosen, who prepared an edition of the Arabic text, expressed a rather critical view of the problems presented in “The Book of Wills,” arguing that many of them were mathematically inconsistent. In his interpretation, such constructions were introduced in order to force solutions that conformed to the inheritance rules applied by Islamic jurists.

According to Rosen, both jurists and the author himself were primarily concerned with protecting heirs and close relatives from claims made by unrelated individuals to whom a dying person might have bequeathed part of his property. He further suggested that restrictions related to the manumission of slaves or the amounts allocated for their redemption were of an artificial nature.

Nineteenth-century studies in the history of mathematics largely neglected this part of al-Khwarizmi’s work. For instance, Hermann Hankel, in his well-known study on the history of mathematics up to 1874, summarized the development of



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science during the so-called “Arab period” but omitted inheritance-related problems. While acknowledging their practical importance in the Muslim world, he considered them to lack scientific interest.

A similar position was taken by Moritz Cantor. After analyzing the first half of al-Khwarizmi’s treatise, he described the remaining portion as a collection of highly complex and obscure problems concerning inheritance law, manumission, and related issues. Cantor emphasized that this section, though significant within its original context, possessed a distinctly “Arabic” character and should be regarded primarily as a foundation for later specialized works dealing with inheritance calculations.

This perspective was reiterated by Julius Ruska in 1917. Likewise, in his 1904 publication on the history of mathematics, Siegmund Günther referred to the content of “The Book of Wills” as a “collection of misunderstandings.”

A more systematic mathematical analysis of these inheritance problems was undertaken by the mathematician and historian Gustav Wertheimer, who published a dedicated study in 1922. He observed that such problems are not easily comprehensible and require considerable effort to master. At the same time, he noted their relevance both for historians of mathematics and for modern educators teaching the theory of fractions. In his work, he provided detailed commentary on nine types of inheritance problems discussed by al-Khwarizmi and concluded that, in the absence of earlier known Arabic sources, al-Khwarizmi may have been the first to establish the tradition of applying algebra to the division of inheritance in accordance with canonical legal norms.

It should be noted, however, that many of these legal principles had already been developed earlier by the prominent jurist Abu Hanifa (690–767). According to Wertheimer, beginning with al-Khwarizmi, “a continuous chain of mathematical treatments of such problems persisted for centuries.” Nevertheless, several important questions remain unresolved: whether al-Khwarizmi relied on earlier sources; what innovations were introduced by his successors; and whether this branch of Arabic algebra was transmitted to the Latin West, and if so, through which channels.

Inheritance relations in the medieval East were primarily grounded in kinship structures that governed the distribution of property. Since enslaved individuals



were legally regarded as the property of their owners, the norms of inheritance law were also extended to slaveholding relations. In this way, algebraic reasoning penetrated deeply into multiple layers of complex social reality.

The dialectical relationship between algebra and the practice of inheritance law can be understood as reciprocal: on the one hand, legal practice provided a strong stimulus for the formation and development of algebra as a scientific discipline; on the other hand, algebra contributed to imposing a certain degree of order and rational structure upon inheritance law, whose foundational norms within Islamic jurisprudence were often highly complex and internally inconsistent.

As noted above, the section entitled “The Book of Wills” constitutes a distinct and substantial part of Muhammad ibn Musa al-Khwarizmi’s treatise. It opens with a chapter on “Ready Money and Debt,” which includes three problems. The first describes a case in which a deceased man leaves two sons, bequeaths one-third of his estate to an unrelated individual, and possesses ten dirhams in cash along with a debt equivalent to one son’s share. The solution proceeds by treating the debt as an unknown quantity added to the known assets, forming a total estate expressed algebraically. After deducting the one-third bequest, the remainder is distributed between the sons.

Al-Khwarizmi formalizes this procedure algorithmically: if the loaned amount is denoted by x , then the total estate equals $10 + x$. Since the shares of the two sons and the beneficiary are equal, one derives a relation that yields $x = 5$. This example illustrates the systematic transformation of a legal problem into an algebraic structure.

Inheritance distribution in Islam is regulated by established legal principles derived from the Qur’anic text, particularly the Qur’an, where heirs are classified into distinct categories: fixed-share heirs (those explicitly assigned portions), agnatic heirs (who inherit the residue), and extended kin entitled to inherit in the absence of the former groups.

In another example, al-Khwarizmi considers a case involving four sons and a relative whose share is defined by a compound condition involving equality with each son’s portion plus an additional fractional adjustment. When translated into modern mathematical notation, the problem can be expressed as an equation relating the total estate and individual shares. Solving this equation demonstrates



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that the total estate corresponds to a specific multiple of the individual share, again highlighting the analytical power of algebraic reasoning in legal contexts. A further example concerns a deceased individual leaving a mother, a wife, two brothers, and two sisters, along with a bequest of one-ninth of the estate. The method involves normalizing the estate into fractional units—initially forty-eight parts—then adjusting the total to account for the bequest, resulting in fifty-four parts. The distribution is then carried out proportionally among the heirs according to their prescribed shares. This procedure reflects a sophisticated use of fractional arithmetic and proportional reasoning.

At the foundation of these problems lie the normative principles of Islamic inheritance law, articulated in the Qur’anic verses of Surah al-Nisa, which define precise shares for heirs under varying familial conditions. These prescriptions require careful calculation, especially when multiple claims and constraints intersect.

In practice, such cases often led to mathematically complex problems, sometimes involving quadratic relationships. Judges (qadis) were not always equipped to resolve them independently and thus turned to specialists in mathematical sciences. In this context, al-Khwarizmi’s algebra—particularly the section on wills—can be regarded as a practical manual designed to address real legal and social challenges.

Over time, the systematic study of inheritance calculations evolved into a distinct discipline known as ‘ilm al-farā’id (the science of inheritance). One of its major later developments is associated with Baha al-Din al-Amili, whose work *Khulasat al-Hisab* represents a significant contribution to this field.

Conclusions

The “The Book of Wills” by Muhammad ibn Musa al-Khwarizmi demonstrates the close interconnection between his scholarship and the socio-legal realities of his time. In this work, the scholar addresses issues of significant relevance to medieval society by presenting methods for resolving legal problems through mathematical models. At the same time, it reveals the conceptual harmony between Islamic legal philosophy and mathematical reasoning.



The treatise systematically explains how matters of inheritance, wills, and financial relations can be resolved on the basis of precise calculation and rational analysis. From this perspective, the work can be regarded as a mathematical substantiation of legal logic and economic thought within the framework of Arab-Islamic civilization.

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