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METHODOLOGICAL RECOMMENDATIONS FOR SOLVING NON-STANDARD EQUATIONS

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Abstract

This article provides a brief analysis of the literature on the topical problem of solving non-standard equations, provides guidelines for teaching schoolchildren to solve non-standard equations and organizing the search for such a solution. In the course of the study and the experience of working with schoolchildren at the seminar "preparing for the exam" for teachers and students in grades 10-11, it was revealed that the greatest difficulty is the search for solutions to non-standard equations containing several functions. According to the author, the use of standard methods does not always allow solving equations of this type. However, the problem of teaching the search for solutions to non-standard equations in the scientific, methodological and educational literature is not sufficiently developed. Numerous problem books contain various examples of solving non-standard equations without their detailed analysis and methodological recommendations for organizing the search for their solution. The author proposes methodological recommendations for teaching students to find solutions to non-standard equations using a system of leading questions and some non-standard methods for their solution, in particular, the assessment method, analytical-functional method and the use of homogeneity.

Keywords: Equation solution, estimation method, domain of definition, domain of values, solution methods.

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Introduction

Practical experience shows that a complex-looking equation often intimidates students, and many of them do not even begin to solve it. Those who do attempt to solve it try unsuccessfully to apply familiar solution methods without analysing the equation itself. Familiarising learners with certain methodological recommendations can help avoid this problem and equip them with specific techniques for solving non-standard equations.

The aim of the study is to develop methodological recommendations for teaching students how to search for solutions of non-standard equations.

Research methods were comprehensive in nature. Among them were:

Theoretical methods: Analysis of scientific-methodological articles and textbooks; analysis of algebra and introductory analysis textbooks and problem books.

Empirical methods: Questionnaires for students and mathematics teachers; observation of the process of searching for solutions to non-standard equations; pedagogical experiment.

An analysis showed that there are a number of studies on teaching how to solve non-standard equations (e.g. works by Yu. M. Kolyagin, I. F. Sharygin, V. I. Golubev, G. V. Dorofeev, G. K. Muravin, A. G. Merzlyak, O. Yu. Cherkasov, A. G. Yakushev, etc.). However, the problem of teaching the search for solutions to non-standard equations in the scientific-methodological and educational literature is not sufficiently developed. Numerous problem collections [1–3] contain various examples of solving non-standard equations without detailed analysis or methodological recommendations for organising the solution search.

One important recommendation from methodologists [4–6] when solving problems is: before solving a problem, begin by analysing the data given in the problem. The process of searching for a solution to non-standard equations is no exception. If hasty actions may sometimes be permissible for standard equations, such haste will not lead to anything good when solving non-standard equations. Not only standard techniques but also non-standard solution methods are necessary.

The literature analysis showed that in methodological literature [7; 8] a non-standard solution method for equations is understood as a method in which the

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properties of functions (monotonicity, evenness, oddness, periodicity, etc.) play the main role when transforming the equation to an equivalent form. L. K. Sadikova [9] and L. S. Kapkaeva [10] highlight functional methods of solving equations and inequalities, the acquaintance with which will allow students to be more successful when solving non-standard equations.

Let us present materials from article [5, p. 214].

- Solve the equation: $\sqrt{5x - x^2 - 6} + 3^{\sqrt{x-\pi}} = \sqrt{1 - 3x}.$

As we see, three different functions are involved in this equation. The application of known methods did not allow students to find a solution independently. Those who began to solve the equation only tried squaring both sides, which of course did not lead to anything good. The presence of an exponential function meant that one could not simplify the equation by squaring. Only with the help of guiding questions, organised during the solution search and analysis of the data, did some learners begin to identify the functions involved and consider the conditions for the existence of the equation's solution, i.e. they started analysing the domain of definition of the equation.

After finding the domain of definition, the learners determined:

а) $\sqrt{5x - x^2 - 6}$ function exists only for x, in the interval [2; 3];

б) $3^{\sqrt{x-\pi}}$ – function exists only for x in the interval [π, +∞);

в) $\sqrt{1 - 3x}$ – function exists only for x in the interval (–∞, 13].

These intervals do not overlap, and therefore the given equation has no solutions.

Answer: No solutions.

1. Solve the equation: $\sqrt{-x^2 + 5x - 6} = 4^{\sqrt{x-3}} - 1$

Encouraged by success after the first equation, most students began by analysing the domain of permissible values. This led to the conclusion that the domain of permissible values consists of only one number, namely x=3. This suggests that if a solution exists, it can only be at x=3. A simple check by substituting x=3 into the equation confirms the final answer: x=3 is a root of the equation.

Answer: x = 3.

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2. Solve the equation: $2sin^2\frac{x}{6}cos^2\frac{x}{12} = x^2 + \frac{1}{x^2}$.

Following similar steps in solving this equation did not yield the expected result, since the domain of definition is all real numbers except 0. However, some students did not rush to apply known methods immediately. After the guiding questions, they came to the conclusion that one can analyse not only the domain of definition but also the range of values of the functions involved in the equation.

By estimating the left-hand side of the equation $2sin^2\frac{x}{6}cos^2\frac{x}{12} \leq 2$, the students concluded that the left side is at most 2. A similar analysis of the right-hand side yields that it is at least 2. The only possible solution is when the left side and the right side are both equal to 2.

Solving the equation $x^2 + \frac{1}{x^2} = 2$ gives roots $x_1 = 1$ and $x_2 = -1$. Substituting these into the original equation shows that neither number satisfies the equation.
Answer: No solutions.

After solving the above, the students were presented with six more equations:
3. $5sin^2x - 2sinxcosx - 3cos^2x = 0$.
4. $3x^2 - 2xy - y^2 = 0$.

5. $4^x - 7*36^x = 18*18^{2x}$.
6. $8y^3 + 11x^2y - y^2x - 18x^3 = 0$.
7. $x + 6 - 4\sqrt{x^2 + 4x - 12} = 8 - 4x$.
8. $5*4^x - 2*2^x log_2 x - 3log_2{}^2 x = 0$.

All these equations are homogeneous in a certain sense, but unfortunately none of the students initially noticed this. Most students solved the first equation, but failed to see that the remaining five equations are also homogeneous. Unsuccessful attempts to apply various solution methods led the students to the idea that they should again begin by analysing the equation. All these equations are homogeneous in a certain sense, but unfortunately none of the students initially noticed this. Most students solved the first equation, but failed to see that

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Leading questions guided the students to discover the similarity among the given equations. In the ensuing discussion of the general form of a homogeneous trigonometric equation and extraction of its solution algorithm, the students arrived at the general form of a homogeneous second-degree equation $Af^2(x) + Bf(x)g(x) + Cg^2(x) = 0$ and its solution method.

After such preliminary work, the students were able to solve the proposed equations.

Consider the solution of equation 4 (from the list above):

Solve: $3x^2 – 2xy – y^2 = 0$.

1) If $y^2 = 0$, $x = 0$, i.e. the point (0, 0) – is a solution of the equation.

2) Now assume $y^2 \neq 0$, and divide both sides of the equation by y^2:

Transform the equation to the form: $3\frac{x^2}{y^2} - 2\frac{xy}{y^2} - \frac{y^2}{y^2} = 0$, and introduce a substitution. Let

$x/y = t$.

Then the equation becomes $3t^2 – 2t – 1 = 0$. The discriminant is $D/4 = 1 + 3 = 4$, so the solutions are $t_1 = 1$, $t_2 = – 1/3$, Returning to the substitution, we get the roots of the equation: $x/y = 1$ или $x/y = – 1/3$.

Answer: $x = y$, $x = – 1/3y$.

Next, consider equation:

Solve: $4^x – 7*36^x – 18*18^{2x} = 0$. Rewrite this as:

$2^{2x} – 7*18^x*2^x – 18*18^{2x} = 0$. Divide both sides by $18^{2x} \neq 0$.

to obtain: $2^{2x}/ 18^{2x} – 7 \cdot 2^x/18^x – 18 = 0$. Thus we get: $(1/9)^{2x} – 7(1/9)^x – 18 = 0$.

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Let t =$(1/9)^x$, with t> 0. Then $t^2 - 7t - 18 = 0$.

By Vieta's formulas, the roots are $t_1 = 9$, $t_2 = -2$ – discard t_2 since t>0.
From (1/9)x = 9, we get x = –1.
Answer: x = –1.

In the discussion, we similarly arrived at the general form of a homogeneous third-degree equation: $Af^3(x) + Bf^2(x)g(x) + Cf(x)g^2(x) + Dg^3(x) = 0$.

The solution of equation $8y^3 + 11x^2y - y^2x - 18x^3 =$ is now straightforward, as
the students have learned the method of solving such equations:

1) If x = 0, then y = 0, i.e. the point (0, 0) is a solution of the equation.
2) Let x ≠ 0, divide both sides by x^3: $8(y/x)^3 + 11y/x - (y/x)^2 - 18 = 0$.
Let $t = y/x$, Then we obtain the cubic equation: $8t^3 - t^2 + 11t - 18 = 0$.
We can guess one root: t=1. Divide the polynomial $8t^3 - t^2 + 11t - 18$ by (t – 1), giving a quadratic: $8t^2 + 7t + 18 = 0$.
Discriminant is D = 49 – 32*18 < 0 – so there are no other real roots.
Thus, t = 1, hence, $y/x = 1$, т.е. y = x.
Answer: y = x.

Consider the equation: 9. $4x^2 + 12x\sqrt{x+1} = 27(1+x)$.
Let $t = \sqrt{x+1}$ Using this substitution, we obtain a homogeneous equation:
$4x^2 + 12xt - 27t2 = 0$.
Since x = –1 is not a root of the equation (because at this value, the variable t becomes
zero), we divide both sides of the equation by t^2: $4(x/t)^2 + 12(x/t) - 27 = 0$.
Let x/t = n, we get the equation $4n^2 + 12n - 27 = 0$.
$D/4 = 36 + 108 = 144$.
$n_1 = (-6+12) / 4 = 3/2$, $n_2 = (-6-12) / 4 = -9/2$.
Let's return to the substitution: x/t = 3/2 or x/t = – 9/2. Hence, taking into account the
substitution,
$x/\sqrt{(x+1)} = 3/2$ (1) or $x/\sqrt{(x+1)} = -9/2$ (2).
Consider equation (1): $x/\sqrt{(x+1)} = 3/2$.

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The roots of this equation are:
$x_1 = 3$ – satisfies the condition $x \geq 0$,
$x_2 = -3/4$ – does not satisfy the condition.
Similarly, we solve equation (2): $x/\sqrt{(x+1)} = -9/2$.
The roots of the equation are: $x_1 = (81 - 9\sqrt{97})/8$ – satisfies the condition $x < 0$,
$x_2 = (81 + 9\sqrt{97})/8$ – does not satisfy the condition.
Answer: $x_1 = 3$, $x_2 = (81 - 9\sqrt{97})/8$.

Functional-Analytic Method

Often, by analysing an equation one can guess its root(s). However, this is not sufficient for solving the equation; an important task is to prove that no other roots
exist. The root theorem (or its corollary) can help with this:
Root Theorem: Let $y = f(x)$ be an increasing (or decreasing) function on a set subset
$D(f)$, and let a be any value taken by $f(x)$ on X. Then the equation $f(x) = a$ has exactly
one root on the set X.
Corollary: If $y = f(x)$ is increasing and $y = g(x)$ is decreasing (or vice versa), then the
equation $f(x) = g(x)$ has at most one root.
Algorithm for solving equations by the functional-analytic method:
- Guess the root;
- Prove that there are no other roots.

Let us consider the following equations:
10. $\sqrt{2x} - 1 + 2^x = \sqrt{10} - x$.
11. $2^x + 3^x + 4^x = (3 + x)5^x$.

Solve equation $\sqrt{2x} - 1 + 2^x = \sqrt{10} - x$. We can guess a root: $x = 1$. On the left-hand
side, we have an increasing function (the sum of two increasing functions and). On the

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right-hand side, we have a decreasing function. By the corollary of the root theorem,
this equation has at most one solution. Since we have found x = 1 as a root, it must be
the only one.
Answer: x = 1.

Solve equation $2^x + 3^x + 4^x = (3 + x)5^x$. We can guess a root: x = 0. Let us prove there
are no other roots. In the given form, one cannot immediately determine the
monotonicity of the functions on each side. Transform the equation by dividing both
sides by 5^x: $(2/5)^x + (3/5)^x + (4/5)^x = 3 + x$. The left-hand side is now a decreasing function of, while the right-hand side is increasing. By the root theorem, there can be
no other roots besides x=0.
Answer: x = 0.

Non-Standard Substitutions
Sometimes it is useful to introduce homogeneity into an equation via a clever substitution. Consider the equation:
12. $(6x+7)^2(3x + 4)(x + 1) = 6$.

Transform the equation by trying to highlight common factors in the multipliers: $(6x+7)^2*1/2(6x + 8)*1/6(6x + 6) = 6$, which simplifies to $(6x+7)^2(6x + 8)(6x + 6) = 72$.
Introduce the substitution 6x + 7 = y, Then 6x+8 = y+1 and 6x+6 = y - 1, so the equation becomes $y^2(y + 1)(y – 1) = 72$, i.e. $y^4 – y^2 – 72 = 0$.
By Vieta's theorem for $y^4 – y^2 – 72 = 0$ we find the quadratic in $y^2 = – 8$ or $y^2 = 9$,
The equation y^2 = -8 has no (real) solutions, whereas y^2 = 9 gives y = 3 or y = -3. Returning to the variable x:
For y = 3: 6x + 7 = 3 ; x = – 2/3.

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For y = -3: 6x + 7 = -3 ; x = – 5/2
Answer: x = –2/3, x = – 5/2.

Conclusion

To summarize everything stated above, we can conclude that when solving non-standard equations, one should not rush. Before attempting to solve an equation, it is useful to perform a detailed analysis of the equation itself: analyze the form of the equation and try to classify it as a known type; identify the functions involved; analyze the domain of definition (domain); estimate the range of values for the left-hand side and the right-hand side of the equation. Only after these steps should one proceed to solve the equation itself. Quite often, these measures alone are sufficient to solve equations that initially appear quite complex.

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