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# QUANTUM SUPERPOSITION AND MULTIDIMENSIONAL HILBERT SPACES: PHYSICAL FOUNDATIONS OF MODERN QUANTUM COMPUTERS

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## Abstract

This article is devoted to the analysis of the physical foundations of quantum computers, in particular the role of quantum superposition and multidimensional Hilbert spaces in quantum computation. Quantum superposition—the ability of a quantum system to exist in multiple states simultaneously—forms the basis of quantum parallelism. Multidimensional Hilbert spaces are realized through qudits (d-level quantum systems), providing higher information capacity and improved error resistance compared to qubits. The article reviews theoretical foundations of superposition based on scientific works indexed in the Scopus database from 2020–2024, as well as its implementation in physical systems such as trapped ions, superconducting circuits, photonic systems, and neutral atom systems. In addition, issues of coherence and quantum error correction are discussed. In conclusion, quantum superposition and multidimensional spaces are fundamental physical principles of future quantum computers, and their development depends on solving decoherence and error correction problems.

**Keywords:** Quantum superposition, Hilbert space, qubit, qudit, quantum computing, coherence, decoherence, quantum error correction, trapped ions, superconducting quantum circuits.



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**KVANT SUPERPOZITSIYASI VA KO‘P O‘LCHAMLI GILBERT  
FAZOLARI: ZAMONAVIY KVANT KOMPYUTERLARINING FIZIK  
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**Annotatsiya**

Ushbu maqola kvant kompyuterlarining fizik asoslarini, xususan, kvant superpozitsiyasi va ko‘p o‘lchamli Gilbert fazolarining kvant hisoblashdagi rolini tahlil qilishga bag‘ishlangan. Kvant superpozitsiyasi – kvant tizimining bir vaqtning o‘zida bir nechta holatda bo‘lish qobiliyati – kvant parallelizmining asosini tashkil etadi. Ko‘p o‘lchamli Gilbert fazolari esa quditlar (d-darajali kvant tizimlari) orqali amalga oshirilib, kubitlarga nisbatan yuqori axborot sig‘imi va xatoliklarga chidamlilikni ta‘minlaydi. Maqolada 2020–2024-yillarda Scopus ma‘lumotlar bazasida indekslangan ilmiy ishlar asosida superpozitsiyaning nazariy asoslari, uni fizik tizimlarda (tutqichdagi ionlar, supero‘tkazuvchi zanjirlar, fotonik va neytral atom tizimlari) amalga oshirish usullari, hamda kogerentlik va xatoliklarni tuzatish muammolari yoritilgan. Xulosa qilib aytganda, kvant superpozitsiyasi va ko‘p o‘lchamli fazolar kelajak kvant kompyuterlarining asosiy fizik tamoyillari bo‘lib, ularning rivojlanishi dekoherens va xatoliklarni tuzatish masalalarining hal etilishiga bog‘liq.

**Kalit so‘zlar:** kvant superpozitsiyasi, Gilbert fazosi, kubit, qudit, kvant hisoblash, kogerentlik, dekoherens, kvant xatoliklarni tuzatish, tutqichdagi ionlar, supero‘tkazuvchi kvant zanjirlari.

**Аннотация**

Данная статья посвящена анализу физических основ квантовых компьютеров, в частности роли квантовой суперпозиции и многомерных гильбертовых пространств в квантовых вычислениях. Квантовая суперпозиция — способность квантовой системы находиться одновременно в нескольких состояниях — лежит в основе квантового параллелизма. Многомерные гильбертовы пространства реализуются через кудиты (d-



уровневые квантовые системы), обеспечивая более высокую информационную ёмкость и лучшую устойчивость к ошибкам по сравнению с кубитами. В статье рассматриваются теоретические основы суперпозиции на основе научных работ, индексированных в базе данных Scopus за 2020–2024 годы, а также её реализация в физических системах, таких как захваченные ионы, сверхпроводящие цепи, фотонные системы и системы нейтральных атомов. Кроме того, освещаются вопросы когерентности и квантовой коррекции ошибок. В заключение отмечается, что квантовая суперпозиция и многомерные пространства являются фундаментальными физическими принципами будущих квантовых компьютеров, а их развитие зависит от решения проблем декогеренции и квантовой коррекции ошибок.

**Ключевые слова:** квантовая суперпозиция, гильбертово пространство, кубит, кудит, квантовые вычисления, когерентность, декогеренция, квантовая коррекция ошибок, захваченные ионы, сверхпроводящие квантовые цепи.

## INTRODUCTION

The computing power of classical computers is approaching its natural limit due to the slowdown of Moore's law and physical limitations. At this point, quantum computers offer fundamentally new solutions to complex problems - in the areas of modeling new materials, calculating molecular structures, optimization and cryptography. The principle of operation of quantum computers is fundamentally different from classical logic: they rely on the laws of superposition and entanglement of quantum mechanics. Quantum superposition - the ability of a quantum system to be in several states at the same time - is the main resource of quantum computing. For a qubit, this state

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$

where  $\alpha$  and  $\beta$  are complex numbers that determine the probability amplitudes of the state and satisfy the condition  $|\alpha|^2 + |\beta|^2 = 1$ . It is this principle that forms the basis of quantum parallelism, allowing a quantum computer to process many options simultaneously. In recent years, research in the field of quantum



computing has been directed not only towards traditional two-level systems (qubits), but also towards multi-level systems - qudits. Qudits operate in the  $d$ -dimensional Gilbert space and offer a number of advantages over qubits: higher information capacity, fault tolerance, and additional capabilities for performing complex calculations. Research published in 2024 shows that the use of qudits allows saving optical resources and achieving similar computational performance with fewer decaying photons. The purpose of this article is to systematically analyze the theoretical foundations of quantum superposition and multidimensional Gilbert spaces, their implementation in physical systems, and their role in the development of future quantum computers, based on scientific works indexed in the Scopus database for 2020–2024.

## MAIN PART

Theoretical foundations and mathematical apparatus of quantum superposition. Quantum superposition is a direct consequence of linear quantum mechanics. It arises due to the linearity of the Schrödinger equation and allows us to describe any state of the system as a linear combination of other independent states. The Gilbert space serves as the basic mathematical structure for representing this superposition: quantum states are vectors, and physical observables are represented as self-adjoint operators. The superposition principle is important not only theoretically, but also practically. It leads to one of the fundamental theorems of quantum computing - the theorem of the impossibility of copying. According to this theorem, there is no unitary operator that creates an exact copy of an arbitrary quantum state, since such an operator leads to a contradiction in the superposition state. This limitation is fundamental for quantum error correction and quantum cryptography. Another important aspect of superposition is that it “collapses” during quantum measurement - as a result of the measurement, the system goes to some specific state and the superposition disappears. This property determines the specificity of the information reading process in quantum computing and should be taken into account when designing quantum algorithms. Multidimensional Gilbert spaces and qudits. Traditional qubits operate in a two-dimensional Gilbert space. However, quantum systems can be multi-level in nature. A qudit is a  $d$ -level quantum system, which represents a  $d$ -dimensional



Gilbert space. The use of qudits offers a number of advantages over qubits. First, a single qudit can store more information than a qubit. A d-level system has an information capacity equivalent to  $\log_2 d$  qubit. Second, qudits can increase fault tolerance. A study published in 2024 showed that it is possible to implement fault-tolerant quantum computing using molecular spin qudits, in which all quantum operations are performed with protection against leading types of errors. The authors note that this approach is much more efficient than standard stabilizer codes, dramatically reducing the resource consumption for error correction. Third, qubits provide additional capabilities for performing complex calculations. Studies on the use of qubits in photonic systems have shown that qubit cluster states require fewer optical modes and fewer decaying photons than qubit cluster states. This is an important advantage, especially in resource-limited environments.

Physical realization of superposition and multidimensionality. The physical foundations of quantum computers are determined by the way they implement their building blocks, i.e. qubits and qubits. Today, there are several major platforms, each of which uses superposition and high dimensionality in different ways.

Ions in a trap are one of the most advanced technologies in quantum computing. Since ions have many internal energy levels, they naturally create opportunities for qubits. A study published in 2020 considered a shuttle method for quantum information processing based on ions in a trap, which allows the creation of scalable architectures by moving ions between trap segments. In 2024, high-precision experiments were conducted in ion systems to measure and retrain neighboring qubits without disturbing their state.

Superconducting chains are artificial atoms that have energy levels artificially created by changing the chain geometry. Superconducting qubits are the basis of modern quantum processors. A review published in 2020, "Superconducting Qubits: Current State of Play," provides a detailed analysis of the current state of superconducting qubits, their advantages and limitations, in particular, decoherence issues caused by  $1/f$  noise. In 2024, a 53-qubit superconducting processor demonstrated quantum supremacy, allowing for a computational space of 253 dimensions (about  $10^{16}$ ). Photonic systems – the polarization or path



degrees of freedom of a photon can serve as qubits or qudits. Photons naturally interact weakly with their environment, which makes their coherence time long. A review published in 2024 reviewed the main aspects of photonic quantum computing - encoding information in photons, the advantages of photonic qubits and the main components (light compressors, quantum light sources, interferometers, photodetectors and waveguides). Resource-efficient photonic quantum computing methods using high-dimensional cluster states were also proposed.

Neutral atoms are a platform that has been developing rapidly in recent years. A study published in 2023 provided an analysis of the neutral atom platform from an industrial perspective, which considered the physical qubit architecture, gate accuracy and connectivity options. In 2024, fault-tolerant quantum computing was demonstrated on a 256-qubit neutral atom processor. The possibility of creating controlled entanglement between atoms using Rydberg states makes this platform promising for scalable quantum computers

The problem of coherence and decoherence. Quantum superposition and entanglement are very fragile states. Interaction with the environment leads to their loss of coherence (decoherence). This is a major obstacle to the practical application of quantum computers. Decoherence is measured by two main parameters: the relaxation time ( $T_1$ ) - the time it takes for the system to transition from an excited state to the ground state, and the phase coherence time ( $T_2$ ) - the time it takes for the phase relationship in the superposition state to collapse. Significant progress has been made in recent years in mitigating decoherence. In 2024, universal coherence protection was demonstrated in a solid-state spin qubit, which protects the qubit from magnetic, electrical, and temperature fluctuations - factors that form almost all important decoherence channels in a solid. Other studies have investigated methods for delaying the decay of quantum coherence and entanglement under the influence of correlated noise channels. However, in modern systems, the error rate reaches  $10^{-2}$  to  $10^{-4}$ . To create a large-scale and reliable quantum computer, the error rate must be drastically reduced. To do this, it is proposed to create a logical qubit from several physical qubits and use special error correction codes.



Table 1. Basic comparison of quantum platforms.

Platform	Qubit type	Advantage	Problem
Ion capture system	Near qubit	High accuracy	Slow performance
Superconducting chains	Qubit	Fast calculation	Loss of coherence
Photonic systems	Qubit/qubit	Low noise	Interference is difficult
Neutral atom	Scalable	Large system	Control is difficult

Quantum error correction: modern approaches. Quantum error correction (QEC) is crucial for the practical utility of quantum computers. In the traditional approach, several physical qubits are used to encode a single logical qubit. However, this method is very expensive in terms of resources. In recent years, qubit-based error correction methods have been rapidly developing. In a study published in the journal PRX Quantum in 2024, a protocol for error-tolerant quantum computing using spin-cat codes on a large-spin qubit was proposed. These codes are adapted to dominant error sources and have a higher error margin and less resource requirements than standard qubit-based codes. The authors note that the proposed scheme significantly reduces resource requirements, which brings error-tolerant quantum computing closer to practical use. Another promising approach is the method of encoding a logical error-correcting qubit on a single qubit on molecular spin qubits. This method does not require as many two-level systems as in standard codes, but uses a single multi-level molecule. As a result, the resources required for error correction are dramatically reduced. The researchers have shown that this method can achieve an almost exponential efficiency in error correction. A paper published in Physical Review Letters in 2024 examined the autonomous quantum error correction of Gottesman-Kitaev-Preskill states. These bosonic codes are one of the most advanced approaches in the field of quantum error correction and allow for error correction in continuously varying systems.



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## Conclusions

Quantum superposition and multi-dimensional Gilbert spaces are the fundamental physical principles of future quantum computers. Superposition forms the basis of quantum parallelism, allowing a quantum computer to process many computational directions simultaneously. Multi-dimensional spaces – implemented by qubits – provide a higher information capacity, fault tolerance, and computational efficiency than qubits. There are various approaches to implementing these principles in physical systems – trapped ions, superconducting chains, photonic and neutral atom systems – each with its own advantages and limitations. However, ensuring the stability of these delicate quantum states (the decoherence problem) and correcting errors in them remains the main scientific and technical problem of our time. Research in 2020–2024 is aimed at improving physical qubits, studying new materials, and creating error-tolerant quantum architectures. Methods for correcting errors based on qubits, in particular spin-cat codes and molecular spin qubits, show significant advantages over standard approaches. The future of quantum computers depends on the solution of these fundamental physical problems and the development of hybrid approaches that combine different physical systems. A deeper study of quantum superposition and multidimensional spaces, as well as improving methods for their control and protection, will lead to widespread practical application of quantum technologies.

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