Modern American Journal of Linguistics, Education, and Pedagogy
ISSN (E): 3067-7874
Volume 01, Issue 03, June, 2025
Website: usajournals.org

SOLVING OLYMPIAD PROBLEMS USING INTEGER AND FRACTIONAL PARTS: A PEDAGOGICAL APPROACH

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Abstract
This article presents a pedagogical framework for solving mathematical Olympiad problems using the integer part ([x], ant'ye) and fractional part ({x}, mantissa) of real numbers. Through detailed solutions to problems from competitions such as Yekaterinburg (2001–2002), SPbGU ITMO (2011–2012), and IMO (1979), we demonstrate techniques involving arithmetic roots, equivalent inequalities, and functional identities. These methods foster creative thinking and analytical skills, making them valuable for mathematics education. The article offers educators and students practical strategies for tackling non-routine problems, enhancing preparation for mathematical competitions. By emphasizing clarity and pedagogical insights, we aim to inspire innovative teaching and learning practices in competitive mathematics.

Keywords: Integer part, fractional part, Olympiad problems, mathematics education, problem-solving, creative thinking

1. Introduction
Mathematical Olympiads challenge students to apply advanced problem-solving techniques and think creatively beyond standard curricula. Problems involving the integer part ($[x]$, the greatest integer less than or equal to x) and fractional

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part ($\{x\} = x - [x]$, where $0 \le \{x\} < 1$) are particularly effective, as they require unconventional approaches and a deep understanding of number properties (Semyenov, 2015). These problems not only test mathematical rigor but also cultivate intellectual curiosity and resilience, making them powerful tools for education.

This article explores the use of ant'ye and mantissa in solving Olympiad problems, with a focus on their pedagogical value. By presenting solutions to carefully selected problems from regional and international competitions, we illustrate methods that engage students and enhance their problem-solving skills. The target audience includes mathematics educators, Olympiad coaches, and students preparing for competitions. Our goal is to provide a framework that bridges mathematical theory and classroom practice, fostering a deeper appreciation for competitive mathematics.

2. Methodology

We selected a diverse set of Olympiad problems that leverage the properties of ant'ye and mantissa. Each problem is solved step-by-step, prioritizing clarity and logical coherence. The solutions rely on key mathematical properties, including:

- $[x] + \{x\} = x$, where $[x]$ is the integer part and $\{x\}$ is the fractional part.
- Inequalities such as $[x] \le x < [x] + 1$.
- Transformations of equations and inequalities into equivalent forms to simplify analysis.

Problems were sourced from reputable competitions (e.g., Yekaterinburg, SPbGU ITMO, IMO) and mathematical archives like Problems.ru and Kvant journal. The solutions are designed to highlight their educational benefits, such as developing analytical reasoning, encouraging creative approaches, and preparing students for the intellectual challenges of Olympiads. Each solution includes a pedagogical insight to guide educators in integrating these problems into their teaching.

3. Results: Selected Problems and Solutions

3.1 Problem 1 (Yekaterinburg, 2001–2002)

Solve the equation

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$[\sqrt{n}+\sqrt{n^2+1}] = [\sqrt{n+10}]$. for natural numbers n.

Solution:

We test small values of n to identify solutions. For n = 2:

$[\sqrt{2}+\sqrt{2^2+1}] = [\sqrt{2}+\sqrt{5}] \approx [1.414+2.236] = [3.65] = 3,$

$[\sqrt{2+10}] = [\sqrt{12}] \approx [3.464] = 3.$

The equation holds. For n = 1 and n = 3, direct substitution shows the equation does not hold. For n ≥ 4, we analyze the inequality:

$\left[\sqrt{n+10}\right] < n < \left[\sqrt{n}+\sqrt{n^2+1}\right].$

- **Left part**: $\sqrt{n+10} < n$. Squaring both sides, $n+10 < n^2$, or $n^2 - n - 10 > 0$, which holds for n ≥ 4 (e.g., for n = 4, 16 – 4 – 10 = 2 > 0).

- **Right part**: $n < \sqrt{n}+\sqrt{n^2+1}$. Since $\sqrt{n^2+1} > n$, the sum $\sqrt{n}+\sqrt{n^2+1} > n$, and its integer part is at least n. Thus, $\left[\sqrt{n}+\sqrt{n^2+1}\right] \geq n > \left[\sqrt{n+10}\right]$ for n ≥ 4, so no solutions exist.

Answer: n = 2.

Pedagogical Insight: This problem teaches students to blend empirical testing with analytical proof, reinforcing their understanding of square roots, floor functions, and inequality manipulation. It encourages a balance between intuition and rigor, a key skill in competitive mathematics.

3.2 Problem 2 (SPbGU ITMO, 2011–2012)

If $[x]\cdot\{x\} = 178$, compute $[x]^2 - [x^2]$.

Solution

Since $[x]\cdot\{x\} = 178$ and the product of an integer and a rational non-integer cannot be an integer, {x} is irrational. Let x = n + p/q, where n = [x], p/q = {x}, and $n \cdot \frac{p}{q} = 178$.

Compute:

$[x]^2 - [x^2] = (n+p/q)^2 - [(n+p/q)^2] =$

$= n^2 + 2n\cdot(p/q) + (p/q)^2 - [n^2 + 2\cdot 178 + (p/q)^2] = n^2 - n^2 - 2\cdot 178 + [(p/q)^2] = -356 + [(p/q)^2]$

Since $0 < p/q < 1, (p/q)^2 < 1$, so $[(p/q)^2] = 0$, yielding –356.

Answer: –356.

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Pedagogical Insight: This problem introduces students to the interplay between integer and fractional parts, encouraging algebraic manipulation and critical thinking about number properties. It fosters precision in handling mixed terms and evaluating floor functions.

3.3 Problem 3 (IMO, 1979)

Prove for all natural n, $n\sqrt{2} > 1/(2n\sqrt{2})$.

Solution:

Let $x = n\sqrt{2}$. We need $\{x\} = x - [x] > 1/(2x)$.

Transform the inequality:

$x - 1/(2x) > [x] \Rightarrow 2n^2 - 1 + 1/(8n^2) > [n\sqrt{2}]^2$.

Since $\left[n\sqrt{2}\right] = \left[\sqrt{2n^2-1}\right]$ and $2n^2 - 1 \geq [\sqrt{2n^2-1}]^2 (as \sqrt{a} \geq [a] \; for\ any\ a)$, the inequality holds for all natural n.

Pedagogical Insight: This problem enhances students' ability to handle irrational numbers and inequalities, promoting precision in algebraic transformations. It encourages exploration of number theory concepts, such as the behavior of irrational multiples.

4. Discussion

The solutions demonstrate the power of ant'ye and mantissa in tackling Olympiad problems, offering students opportunities to engage with non-routine challenges. These techniques encourage creative problem-solving by requiring students to think beyond standard algorithms. For example, Problem 1 combines empirical testing with inequality analysis, Problem 2 emphasizes algebraic manipulation, and Problem 3 introduces rigorous proof techniques for irrational numbers.

From a pedagogical perspective, these problems:

- **Enhance Analytical Skills**: Students learn to break down complex problems into manageable steps, a critical skill for mathematics and beyond.
- **Foster Creativity**: The unconventional use of ant'ye and mantissa encourages students to explore alternative solution paths.

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- **Prepare for Competitions**: Exposure to Olympiad-style problems builds confidence and familiarity with competitive formats.

Educators can integrate these problems into the classroom to stimulate discussion, encourage collaborative problem-solving, and highlight the beauty of mathematical reasoning. By emphasizing the process of discovery, teachers can inspire students to approach mathematics as an intellectual adventure.

5. Conclusion

This article showcases the use of integer and fractional parts in solving Olympiad problems, blending mathematical rigor with pedagogical value. The presented solutions offer clear, step-by-step approaches that educators can use to engage students and enhance their problem-solving skills. By fostering creativity and analytical thinking, these methods prepare students for mathematical competitions and deepen their appreciation for the subject. Future work could explore additional applications of ant'ye and mantissa in areas like number theory, functional equations, or computational mathematics, further enriching mathematics education.

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