Modern American Journal of Linguistics, Education, and Pedagogy
ISSN (E): 3067-7874
Volume 01, Issue 03, June, 2025
Website: usajournals.org

SOLVING CUBIC EQUATIONS USING CARDANO'S METHOD WITH SOFTWARE

Abdushukur Gapparovich Abdurakhmanov
Senior Lecturer Chirchik State Pedagogical University,
Chirchik, Uzbekistan,
abdushukur1969@mail.ru

Abstract
This thesis is devoted to the study of the method for solving cubic equations using Cardano's formula. The thesis provides a detailed algorithm for solving a third-degree equation using this method, and describes working with software for solving cubic equations.

Keywords: Algorithm solution, 3rd-degree equation, software.

1 Introduction

Cubic equations are third-degree equations of the form:
$ax^3+bx^2+cx+d=0$, $a\neq0$
Solving such equations is an important part of algebra and mathematical analysis courses. One of the first effective methods for their solution was proposed by the Italian mathematician Gerolamo Cardano in the 16th century. His work *Ars Magna* (1545) marked a turning point in the history of algebra, where he presented a way to find the roots of cubic equations using radicals.
In modern times, this method has not lost its value: thanks to software, it can be applied quickly and accurately even for complex equations.

2 Reduction to Canonical Form

The general form of a cubic equation is:
$ax^3+bx^2+cx+d=0$
First, we divide the equation by the coefficient a to bring it to a normalized form:
$x^3+(b/a)\ x^2+(c/a)x+d/a=0$
Then, a variable substitution is made:

Modern American Journal of Linguistics, Education, and Pedagogy
ISSN (E): 3067-7874
Volume 01, Issue 03, June, 2025
Website: usajournals.org

x=y−b/3a

The purpose of this substitution is to eliminate the x2 term. As a result, the equation takes on the canonical form:

$y^3+py+q=0$

where:

$$p = \frac{3ac - b^2}{3a^2}, \quad q = \frac{2b^3 - 9abc + 27a^2d}{27a^3}$$

3 Discriminant and Nature of Roots

The discriminant of the equation is used for root analysis:

$\Delta=(q/2)^2+(p/3)^3$

Three cases are considered:

- If $\Delta>0$ – one real root and two complex conjugate roots.
- If $\Delta=0$ – three real roots, two of which are identical.
- If $\Delta<0$ – three distinct real roots.

4 Cardano's Formulas

If $\Delta>0$, the roots of the equation are found by the following formulas:

$$y = \sqrt[3]{-\frac{q}{2} + \sqrt{\Delta}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\Delta}}$$

If $\Delta<0$, trigonometric formulas or complex roots of cubic roots are used, which makes manual calculations cumbersome — this is where software becomes especially useful.

5 Example of Solution Using Software

Consider the equation:

$x^3+3x^2+4x+2=0$

Step 1: Reduce to canonical form using the substitution:

x=y−3/3=y−1

Substituting into the original equation and simplifying, we get:

$y^3+py+q=0$, p=1,q=2

Calculate the discriminant:

Δ=(2/2)2+(1/3)3=1+1/27>0

Modern American Journal of Linguistics, Education, and Pedagogy
ISSN (E): 3067-7874
Volume 01, Issue 03, June, 2025
Website: usajournals.org

one real and two complex roots.
Using software (for example, Python with the SymPy library or WolframAlpha) we get:
$y_1=0$, $y_2=i$, $y_3=-i$
Returning to the variable x:
$x_1=y_1-1=-1, x_2=i-1, x_3=-i-1$
Answer: $x=-1,-1+i,-1-i$

6 Conclusion

Cardano's method remains a powerful and systematic tool for solving cubic equations. Despite the complexity of manual calculations in the general case, modern mathematical software allows for quickly obtaining accurate results, analyzing roots, and visualizing them in the complex plane. The use of programs like Mathematica, Matlab, Maple, SageMath, or Python (SymPy, NumPy) makes learning and applying the method more accessible and illustrative for students in technical and mathematical fields.

7 Student Reflections

Studying Cardano's method, I realized how important it is to understand not only the mathematical algorithms themselves but also their historical and practical significance. This method, developed over 400 years ago, shows how a complex equation can be transformed step by step into a sequence of understandable operations. It is especially interesting to see how ancient approaches are integrated into modern technologies.
Software, such as Python or WolframAlpha, allows us not only to find the roots of equations in a matter of seconds but also to delve deeper into the logic of the method, trace all intermediate steps, and visualize the result. This eliminates routine calculations and leaves more room for analysis and understanding the essence of mathematical processes.
For me as a student, this is especially valuable — I learn not just to "get the answer," but to understand where it comes from. Mastering Cardano's method is not only a step towards better knowledge of algebra but also the development of mathematical thinking and confidence in using digital tools in learning.

Modern American Journal of Linguistics, Education, and Pedagogy
ISSN (E): 3067-7874
Volume 01, Issue 03, June, 2025
Website: usajournals.org

REFERENCES

1. S. Winkler, A. K¨orner, V. Urbonaite/ Maple T.A. in Engineering Educations.// February 2012. IFAC Proceedings Volumes 45(2): 906-911 DOI:10.3182/20120215-3-AT-3016.00160

2 Benty.D.D.N et al.,/"Use of Information and Communication Technology in Learning in the Covid-19 Pandemic Period to Improve Student Learning Outcomes,"// 2020 6th International Conference on Education and Technology (ICET), Malang, Indonesia, 2020, pp. 165-169, doi: 10.1109/ICET51153.2020.9276617

3 Abdurakhmanov,A, Yusupov,M/ The use of mathematical packages in teaching higher mathematics using the example of mathematical analysis.// American Institute of Physics. AIP Conference Proceedings, Volume 3244, Issue 1, id.030053, 8 pp.

4 Tevs D.P., Degtyareva Yu.V/ The use of mathematical packages in the educational activities of students and professional activities of teaching staff // Pedagogical University Bulletin of Altai. - Barnaul. - 2004. - No. 1.

5 N.V. Zaitseva, E.S. Ulyanova.Application of SCM Maple in studying some sections of complex analysis International scientific and practical conference"Information technologies in education and science"(ITON-2016) (November 5-7, 2016, Kazan) pp 27-33.

6 Tuychiev T.T., Tishabaev Zh.K., Dzhumabaev D.Kh., Kitmanov / Independent work on the theory of complex variable functionals// Tashkent. "Mumtoz suz", 2018.

7 Abdurahmanov.A.G. /The use of modern information technology in solving non-standard problems.// *European Journal of Research and Reflection in Educational Sciences Vol* 8.12 (2020).

8 Tamur, M., Juandi, D., & Kusumah, Y. S. (2020). /The Effectiveness of the Application of Mathematical Software in Indonesia//; A Meta-Analysis Study. International Journal of Instruction, 13(4), 867-884. https://doi.org/10.29333/iji.2020.13453

9 Setiawan, A. A., Muhtadi, A., & Hukom, J. (2022). /Blended learning and student mathematics ability in Indonesia//: A meta-analysis study. International Journal of Instruction, 15(2), 905-916. https://doi.org/10.29333/iji.2022.15249a

Modern American Journal of Linguistics, Education, and Pedagogy
ISSN (E): 3067-7874
Volume 01, Issue 03, June, 2025
Website: usajournals.org

10 Mukhamedieva, D.T., Raupova, M.H./ Model for forest ecosystems based on quantum optimization . //E3S Web of Conferences. 2024, 498, 02006.

11Fuchs, K. J. (2001). /Computer Algebra Systems in Mathematics Education. //ZDM The International Journal on Mathematics Education, 35(1), 20–23.

12 Melis, E.; Andr`es, E.; Budenbender, J.; Frischauf, A.; Goguadze, G.; Libbrecht, P.; Pollet, V.; Ullrich, G. /Active Math: A Generic and Adaptive Web-Based Learning Environment. In Proceedings of the Workshop on Emotional and Cognitive// Issues at the International Conference on Intelligent Tutoring System, Montreal, QC, Canada, 23–27 June 2008. [**Google Scholar**]

13 Hui Xu. /Using Mathematical Software in High School Math Class: A Case Study.// International Journal of Information and Education Technology, Vol. 6, No. 12, December 2016.

14 Kuchkarova D.Sh;EXAMPLES OF INCREASE AND DECREASE OF FUNCTION https://webofjournals.com/index.php/4/article/view/2169/214981-89-betlarVolume 2, Issue 11, November 2024 ISSN (E): 2938-3757

15Kuchkarova D.Sh; Computer science and engineering technologies. Collection of materials of the international scientific and technical conference - Jizzakh: “Formation of initial skills in future mathematicians”. Jizzakh branch of the National University of Uzbekistan, October 18-19, 2024. pp. 166-170.

16 Eraliyeva Maqpal Madali qizi 1st year master's student of Tashkent region ChDPU https://doi.org/10.5281/zenodo.8046878 THE PROGRESS OF SCIENCE JOURNAL "TEACHING A SPECIAL TECHNOLOGY OF TEACHING THE HIGHER-ORDER DIFFERENTIAL EQUATIONS SECTION OF THE SCIENCE OF DIFFERENTIAL EQUATIONS"

17Eraliyeva Maqpal Madali qizi 2nd year master's student of Tashkent region ChDPU https://doi.org/10.5281/zenodo.14214401; "SCIENTIFIC FOUNDATIONS OF RAISING THE USE OF INFORMATION TECHNOLOGIES TO A NEW LEVEL AND MODERN PROBLEMS OF AUTOMATION" III-INTERNATIONAL SCIENTIFIC CONFERENCE NOVEMBER 20, 2024 "TEACHING HIGH-ORDER DIFFERENTIAL EQUATIONS USING SPECIAL TECHNOLOGIES"

Modern American Journal of Linguistics, Education, and Pedagogy
ISSN (E): 3067-7874
Volume 01, Issue 03, June, 2025
Website: usajournals.org

18Dilnovaz Kuchkarova Department of Algebra and Mathematical Analysis Chirchik State Pedagogical University; THE IMPACT OF THE RIEMANN MAPPING THEOREM ON ANALYTIC STRUCTURE IN THE THEORY OF COMPLEX FUNCTIONS EduVision: Journal of Innovations in Pedagogy and Educational Advancements Volume 01, Issue 06, June, 2025 brightmindpublishing.com ISSN (E): 3061-6972 Licensed under CC BY 4.0 a Creative Commons Attribution 4.0 International License. https://brightmindpublishing.com/index.php/ev/article/view/1112/1137.